

Multi-Dimensional Passive Sampled Port-Hamiltonian Systems

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Abstract—Passivity of virtual environments running in discrete time is a sufficient condition for stability of the system. The framework for passive sampled Port-Hamiltonian systems allows multi-dimensional virtual environments exhibiting internal dynamic behavior to be computed on a discrete medium in a passive manner. It is shown that a causality analysis is required in the framework to detect if any of the model elements have, a time dependent change of energy function in the energy balance of the system. The Standard Linear Solid model, which is often used to simulate the visco-elastic interaction with soft biological tissue is used as an example. Simulated and experimental results are provided to demonstrate the benefit of the described framework. It is shown that using this approach a multi-dimensional model which is passive in the continuous domain remains passive in the discrete domain, whereas a standard discretization approach can become non-passive.

I. INTRODUCTION

A haptic interface couples the user bidirectionally to a virtual environment. The user can manipulate the objects in the virtual environment by means of this interface. The virtual environment conveys force information about this interaction through the interface to the user. A major research topic in haptics remains the stability of the coupled system [7], composed of the user, the interface, and the virtual environment.

Due to the bidirectional coupling, stability issues can arise. A well known source for possible instability of the system is the interconnection of the continuous and discrete domains through the haptic interface. The sample and Zero Order Hold (ZOH) operations used to connect these two domains can lead to the generation of “virtual” energy. This “virtual” energy can lead to instability of the system [1],[6].

As the generation of this “virtual” energy is a cause for stability problems, a logical step is to apply a control architecture that prevents this energy from being generated. This works especially well for impedance type haptic interfaces. An impedance type haptic interface has a velocity as input and force/torque as output causality seen from the viewpoint of the device. For this type of interface the energy exchange between the continuous and discrete domains can be exactly determined [14]. Such a system is depicted in Fig. 1, where τ_* indicates a force and q_* indicates a position. The subscript $*$ is u or r representing the user and actuators, respectively.

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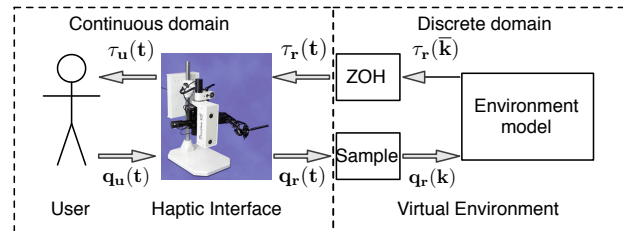


Fig. 1. User interacting with an impedance type haptic feedback system (Freedom 6S [10]) in the continuous domain.

A realistic simulation of the haptic interaction with physical objects is desired. Most often this will involve a complex multi-dimensional virtual environment, e.g. for the haptic interaction with soft biological tissues. A multi-dimensional system is defined as a system containing more than one energy storing element. Energy exchange occurs between the user and the virtual environment, and also between the energy storing elements themselves. If we monitor these energy flows we can determine precisely where, when, and the quantity of “virtual” energy being generated and take measures to ensure passivity of the system is retained.

Stramigioli et al. [15] introduced a framework for monitoring these energy flows and presented a “bookkeeping” algorithm to ensure passivity. This framework however has sometimes been criticized in literature as being unclear on how to deal with multi-dimensional virtual environments [9]. In this paper we show that the framework can accommodate such multi-dimensional systems. With respect to the work presented by Franken et al. [3] a causality analysis is included in the framework and model elements with an admittance causality obtain a time dependent energetic description.

The paper is organized as follows: Section II will place the work into context with different passivity-based approaches. Section III summarizes the theory behind Port-Hamiltonian systems and the framework for sampled passivity. Section IV shows how to handle multi-dimensional systems. Based on an example it is shown that the formulation presented in [3] poses a problem for models which include elements with an admittance causality. For these elements an alternative sample period dependent energetic formulation is presented. Section V presents simulated and experimental results which were obtained with the extended framework. The paper concludes and provides directions for future work in Section VI.

II. PASSIVITY BASED CONTROL ARCHITECTURES

In the previous section we introduced that the generation of “virtual” energy is a possible source for stability problems.

Preventing this “virtual” energy from being generated, or taking adequate measures to dissipate it when it is being generated will guarantee passivity, and therefore stability of the system. A system is said to be passive when the following energy balance holds

$$\int_{t_0}^{t_1} \tau_u(t) \dot{q}_u(t) dt \geq E(0), \quad (1)$$

where $\dot{q}_u(t)$ is the velocity of the user, $\tau_u(t)$ is the feedback force applied by the system to the user, and $E(0)$ is the energy stored initially in the system. $E(0)$ is assumed to be zero. Due to mechanical friction some energy dissipation is bound to occur in the interface. Gillespie et al. [6] presented an analysis of the maximum stiffness that can be implemented passively given a certain sample frequency and the viscous friction in the device. Kim et al. [8] used a similar principle to introduce an energy bounding algorithm which determines the maximum allowable increase in the feedback force at each sample instant in order to limit the generated “virtual” energy to the energy dissipated in the haptic interface and the human operator. In order to derive to a less conservative implementation, they also included an estimation of the damping of the operator’s arm in the analysis.

For impedance type displays the energy which is exchanged between the continuous and discrete domains can be determined precisely by a position measurement as [14]

$$\begin{aligned} \Delta H_I(k) &= - \int_{(k-1)T}^{kT} \tau_r(t) \dot{q}_r(t) dt \\ &= -\tau_r(\bar{k}) \int_{(k-1)T}^{kT} \dot{q}_r(t) dt \\ &= -\tau_r(\bar{k})(q_r(k) - q_r(k-1)), \end{aligned} \quad (2)$$

where $\Delta H_I(k)$ is the exchanged energy, and T is the sample interval. The indices k and \bar{k} refer to variables in the discrete domain. Variables with index k are associated with the k^{th} sample instant and variables with index \bar{k} are associated with the time interval between sample instants $k-1$ and k [3]. If we neglect the energy dissipation and inertial effects of the haptic interface, since they are assumed to be small, it is possible to reformulate (1) using (2) and piecewise integration as

$$\begin{aligned} \int_{t_0}^{nT} \tau_r(t) \dot{q}_r(t) dt &\geq 0 \\ \Rightarrow \sum_{k=1}^n \tau_r(\bar{k}) \int_{(k-1)T}^{kT} \dot{q}_r(t) dt &\geq 0 \\ \Rightarrow \sum_{k=1}^n \tau_r(\bar{k})(q_r(k) - q_r(k-1)) &\geq 0, \end{aligned} \quad (3)$$

where n is the number of samples. Ryu et al. [12] used (2) and (3) to implement a time domain passivity control architecture that monitors the energy exchange between the continuous and discrete domains. They used a viscous damper, modulated by the amount of “virtual” energy generated, to enforce a neutral energy balance according to (3).

All three approaches listed above only take into account the “virtual” energy generated by the sample and ZOH operation. Virtual environments representing physical objects will most likely contain dissipative elements. This means that for those systems a stronger condition of passivity is desired in order to accurately capture the energetic behavior of the system. With respect to systems with internal dissipation we would like to have

$$\sum_{k=0}^n \tau_r(\bar{k})(q_r(k) - q_r(k-1)) \geq H_R, \quad (4)$$

where H_R is the dissipated energy inside the virtual environment. Therefore, the approaches mentioned above, whilst overall passivity is maintained, do not necessarily capture the energetic behavior of the object being modeled by the virtual environment.

III. PASSIVE SAMPLED PORT-HAMILTONIAN SYSTEMS

In the previous section it was discussed that in order to accurately display the energetic behavior of the system we need to monitor the flow of energy through the model. Port-Hamiltonian modeling is centered around the conservation of energy and shares many characteristics with bond graph theory [11]. Every physical system can be described as a combination of energy converting, energy storing, and/or energy dissipating elements which are connected by means of a power preserving structure. Each element is connected to this structure by means of a power port through which energy exchange can take place. This port is described by two variables whose product is power, e.g. forces and velocities for elements from the mechanical domain. The behavior of each element is described by a constitutive relation which relates the two power port variables to each other. The causality of an element (which of the power port variables is an input to the element and which acts as an output) is defined by the collection of elements present in the system and the manner in which they are connected. A Port-Hamiltonian system is composed of a state manifold χ , an energy function $H : x \rightarrow \mathbb{R}$ which expresses the total energy present in the system as function of the state x , and a passive state dependant network structure $D(x)$, called a Dirac structure, which describes how the elements are connected. A thorough explanation of Port-Hamiltonian modeling can be found in [13].

It is well known that a direct discrete implementation of the continuous domain description of models can lead to the generation of “virtual” energy and that this is no different for Port-Hamiltonian systems [15]. However, as the Port-Hamiltonian framework is centered around energy exchange it is possible to implement a passive version of such models by taking the energy distribution into account. The approach is that at each sample instant the energy exchange between the continuous and discrete domains is determined according to (2). This exchanged energy, the previous state of the model, and the Dirac structure are then used to determine the new energy distribution and thus a new state. It should be noted that this redistribution of energy is performed

a posteriori. The updated energy distribution is eventually used to determine the appropriate force for the next sample interval based on the model structure.

As an energy based connection is used between the continuous and discrete domain, the state of the system on either side of this connection can differ as otherwise the connection would not be passive. This is an important aspect of passive sampled Port-Hamiltonian systems. As an illustration we will consider a pure linear spring as virtual environment. Assume that this spring is compressed to state $x(i-1)$ and as a result an opposing force $F = -k_s x(i-1)$ is applied to the user. The user then further compresses the spring by an amount Δq . As given by (2) the user has injected the following amount of energy, ΔH_I , into the virtual environment

$$\Delta H_I = k_s x(i-1) \Delta q \quad (5)$$

This energy is then added to the energy, $H_s(i-1)$ already stored in the spring

$$H_s(i) = H_s(i-1) + \Delta H_I \quad (6)$$

The new state $x(i)$ of the spring follows from the storage function of the spring

$$x(i) = \sqrt{\frac{2H_s(i)}{k_s}} \quad (7)$$

The state jump that has occurred in the discrete system is thus

$$\begin{aligned} \Delta x(i) &= x(i) - x(i-1) \\ &= \sqrt{x(i-1)^2 + 2x(i-1)\Delta q} - x(i-1) \quad (8) \\ &\neq \Delta q \end{aligned}$$

At first glance it might be assumed that contact between the user and the virtual spring can be lost if the discrete state jump is not equal to the physical displacement of the user. In the framework of passive sampled Port-Hamiltonian systems however the state of the virtual spring relates not to a physical position, but to its energetic content. As long as energy is present in the virtual spring in the example above, the connection between the user and the virtual spring is maintained. As energetic interaction is at the center of this framework, the system will be initially in a deadlock situation as no energy is exchanged. How to handle such a deadlock situation is described in [15], [3] and in Section V-A.1.

The key aspect to this approach is to regard a discrete domain Port-Hamiltonian system as a continuous domain Port-Hamiltonian system in which the efforts, e.g. forces for mechanical systems are kept constant during the sample periods and afterwards compute the new energy distribution. This corresponds to the causality of the impedance type haptic interface. This framework was first introduced in [15]. Franken et al. [3] showed that for the well-known Voigt contact model [5], the dissipated energy was handled incorrectly. In the original algorithm the dissipated energy was fixed *a priori* and a new state x was computed based on the resulting change in stored energy. The alteration introduced by Franken et al. [3] was to make the dissipated energy dependent on the state jump Δx of the energy storing

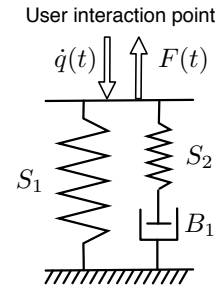


Fig. 2. The Standard Linear Solid (SLS) model.

element and compute the state jump that satisfies the passive energy balance. A further refinement however is needed in order to implement arbitrary multi-dimensional systems. This will be explained in the next section.

IV. MULTI-DIMENSIONAL SYSTEMS

As mentioned in Section I, virtual environments that describe physical objects in a realistic manner are usually multi-dimensional systems. A multi-dimensional system is a system containing more than one energy storing element, e.g. in the mechanical domain either an inertia or a spring. An example of such a multi-dimensional system is shown in Fig. 2. This represents the Standard Linear Solid (SLS) model in the continuous domain, which is often used to describe the visco-elastic response of soft biological tissue [5]. It consists of a spring S_1 with stiffness k_1 parallel to a spring S_2 with stiffness k_2 . S_2 is in series with a viscous damper B_1 with damping coefficient b_1 . Multi-dimensional systems can usually exhibit internal dynamic behavior. For the SLS model, this is the relaxation effect, which means that when a constant strain is applied to the model, the corresponding stresses inside the model decrease over time [5]. This demonstrates that even when the user and the virtual environment are not exchanging energy, energy can be exchanged between the model elements.

Multi-dimensional Port-Hamiltonian systems can be regarded as a collection of Port-Hamiltonian subsystems which are connected by means of a power bond. The SLS for instance can be seen as a spring parallel to a Port-Hamiltonian subsystem. This second Port-Hamiltonian subsystem consists of a spring in series with a damper, Fig. 3. Based on the state of the system a certain amount of energy will be exchanged between the various subsystems. These connecting power bonds will therefore be part of the energy balance of each subsystem. For instance, assume that based on the state of a particular subsystem some amount of energy is sent to a different subsystem. The energy balance of that second subsystem determines how the incoming energy is distributed. Note that the energy balance of this second subsystem can also contain a power bond connecting it to yet another subsystem. Therefore, implementing multi-dimensional Port-Hamiltonian systems comes down to finding the hierarchy in which the various energy balances will have to be solved. As the energy exchange between the user and the virtual environment is known the energy balance of the subsystem to which the user is connected is the first to be evaluated. Then

the energy balances of the subsystems which are connected to the first subsystem are solved and so on. We will show in the following that there can exist elements and even whole collections of elements for which the energy balance is only a function of the state of the model and the duration of the sample period. The SLS will be used as an example to show that the algorithm discussed in [3] needs to be extended to account for those model elements and to display the correct behavior of the virtual environment. For this, we will first follow the steps in the discrete domain as listed in [3].

Assume again that the user has pushed the haptic interface a distance $\Delta q(k)$ against an opposing force $F(\bar{k})$ during the sample interval \bar{k} . The force $F(\bar{k})$ consists of a force $F_1(\bar{k})$, due to spring S_1 , and $F_2(\bar{k})$ due to spring S_2 , and damper B_1 . By doing so the user has injected an amount of energy $\Delta H_I(k)$

$$\Delta H_I(k) = -F(\bar{k})\Delta q(k), \quad (9)$$

into the virtual environment. Part of the energy $\Delta H_I(k)$ is stored in S_1 , but also some energy will flow into the second branch. Spring S_1 will exhibit a discrete state jump of $\Delta x_{s_1}(k)$ associated with the increase of its stored energy $\Delta H_{s_1}(k)$. It is important to note that $\Delta q(k) \neq \Delta x_{s_1}(k)$ due to the fact that we use an energetic description for spring S_1 .

As spring S_1 and the second branch move in parallel, the discrete displacement over the power port connecting the second branch to the user interaction point will be equal to the discrete state jump $\Delta x_{s_1}(k)$. Therefore the energy that flows into the second branch, $\Delta H_2(k)$, will be

$$\Delta H_2(k) = -F_2(\bar{k})\Delta x_{s_1}(k). \quad (10)$$

The increase, $\Delta H_{s_1}(k)$ of energy stored in spring S_1 is linked to its state, $x_{s_1}(\bar{k})$, and the discrete state jump, $\Delta x_{s_1}(k)$ as

$$\begin{aligned} H_{s_1}(\bar{k}) &= \frac{1}{2}k_1(x_{s_1}(\bar{k}))^2 \\ H_{s_1}(\bar{k}+1) &= \frac{1}{2}k_1(x_{s_1}(\bar{k}) + \Delta x_{s_1}(k))^2. \\ \Rightarrow \Delta H_{s_1}(k) &= H_{s_1}(\bar{k}+1) - H_{s_1}(\bar{k}) \\ &= \frac{1}{2}k_1(\Delta x_{s_1}(k))^2 + k_1x_{s_1}(\bar{k})\Delta x_{s_1}(k), \end{aligned} \quad (11)$$

where $H_{s_1}(k)$ is the potential energy storage function of a linear spring. $\Delta H_{s_1}(k)$ is also defined by the distribution of $\Delta H_I(k)$ over $\Delta H_2(k)$ and $\Delta H_{s_1}(k)$ as

$$\Delta H_{s_1}(k) = \Delta H_I(k) - \Delta H_2(k). \quad (12)$$

Combining (11) and (12), and substituting (10) results in a second order equation which can be solved for $\Delta x_{s_1}(k)$. This is graphically depicted in Fig. 3.

The energy $\Delta H_2(k)$ now needs to be distributed in the second branch. According to [3] the dissipated energy is a function of the force exerted by the damper during the sample period multiplied by the displacement associated with its power port assuming a constant velocity. For damper B_1 the constant velocity $v_{B_1}(\bar{k})$ of its power port can be expressed

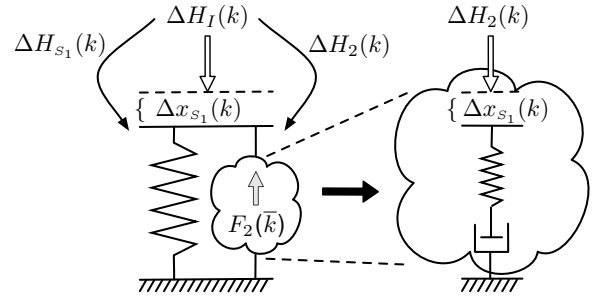


Fig. 3. A multi-dimensional system, consisting of two storage elements, represented as an interconnection of two subsystems.

in terms of the discrete state jumps of springs S_1 , $\Delta x_{s_1}(k)$ and S_2 , $\Delta x_{s_2}(k)$ as

$$v_{B_1}(\bar{k}) = \frac{\Delta x_{s_1}(k) - \Delta x_{s_2}(k)}{T}. \quad (13)$$

Using (13) the energy dissipated by damper B_1 , $\Delta H_{B_1}(k)$ can be expressed as

$$\begin{aligned} \Delta H_{B_1}(k) &= -F_2(\bar{k})v_{B_1}(\bar{k})T \\ &= -F_2(\bar{k})(\Delta x_{s_1}(k) - \Delta x_{s_2}(k)). \end{aligned} \quad (14)$$

As for spring S_1 , an energy balance can be composed in the form of a second order equation. The increase, $\Delta H_{s_2}(k)$ of the energy stored in spring S_2 can be written in the forms of (11) and (12) as

$$\Delta H_{s_2}(k) = \frac{1}{2}k_2(\Delta x_{s_2}(k))^2 + k_2x_{s_2}(\bar{k})\Delta x_{s_2}(k), \quad (15)$$

and

$$\Delta H_{s_2}(k) = \Delta H_2(k) - \Delta H_{B_1}(k). \quad (16)$$

However, the combination of (14) and (16) results in

$$\Delta H_{s_2}(k) = -F_2(\bar{k})\Delta x_{s_2}(k). \quad (17)$$

Combining (15) and (17) and simplifying with respect to $\Delta x_{s_2}(k)$ results in

$$\frac{1}{2}k_2(\Delta x_{s_2}(k))^2 + (k_2x_{s_2}(\bar{k}) + F_2(\bar{k}))\Delta x_{s_2}(k) = 0. \quad (18)$$

Further

$$F_2(\bar{k}) = -k_2x_{s_2}(\bar{k}). \quad (19)$$

Substituting (19) into (18) it always follows that

$$\Delta x_{s_2}(k) = 0. \quad (20)$$

By applying the definition proposed in [3] for the energy dissipated by damper B_1 , spring S_2 is effectively removed from the model, as shown by (20).

In order to find the source of this problem we explicitly derive the continuous domain Port-Hamiltonian description of the system. We first write down the energy function $H(t)$ as

$$H(t) = \frac{1}{2}k_1x_{s_1}(t)^2 + \frac{1}{2}k_2x_{s_2}(t)^2 \quad (21)$$

The forces exerted by springs S_1 and S_2 are given by $\frac{\partial H(t)}{\partial x_{s_1}}$ and $\frac{\partial H(t)}{\partial x_{s_2}}$, respectively. Recall that each element is connected in the model to the network structure by means of a power port. One of the variables of this power port acts

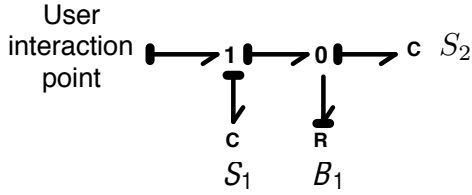


Fig. 4. Bond graph of the Standard Linear Solid (SLS) model showing an admittance causality for damper B_1 .

as an input and the other as an output to the network. The network structure specifies how the input of each element is connected to the outputs of the other elements. If we represent damper B_1 as a power port of which F_{B_1} indicates the force and v_{B_1} the velocity, the system is fully described by the following structure using (21) as

$$\begin{pmatrix} \dot{x}_{S_1} \\ \dot{x}_{S_2} \\ F_{B_1} \\ F \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{\partial H}{\partial x_{S_1}} \\ \frac{\partial H}{\partial x_{S_2}} \\ v_{B_1} \\ \dot{q} \end{pmatrix}, \quad (22)$$

where we have dropped the time dependency for notational simplicity. The equivalent bond graph is depicted in Fig. 4. At a “1”-junction the efforts are additive and the flow constant, and at a “0”-junction this is opposite. The direction of the causal stroke on each power bond gives the direction of the effort variable, e.g. a causal stroke at the element side of a power bond means that the effort is the input and the element computes the resulting flow.

Both the Port-Hamiltonian description given in (22) and Fig. 4 show that damper B_1 has an admittance causality. The force imposed on it by spring S_2 will result in a certain velocity with which the damper is pushed away. Recall that due to the impedance causality of the interface we defined a discrete-time Port-Hamiltonian system as a continuous-time system in which the forces were fixed during the time period between two sample instants. The energy dissipated by the damper will therefore have to be a function of the duration of the sample period. Using this admittance causality, the constitutive relation of a viscous damper, and the duration of the sample period the dissipated energy can be expressed as

$$\begin{aligned} F_{B_1}(\bar{k}) &= \frac{\partial H(\bar{k})}{\partial x_{S_2}}(\bar{k}) \\ &= k_2 x_{S_2}(\bar{k}) \\ v_{B_1}(\bar{k}) &= \frac{F_{B_1}(\bar{k})}{b_1} \\ \Delta H_{B_1}(k) &= F_{B_1}(\bar{k}) v_{B_1}(\bar{k}) T \\ &= \frac{(F_{B_1}(\bar{k}))^2 T}{b_1}. \end{aligned} \quad (23)$$

This time dependency will occur in all parts of the model that have an admittance causality, e.g. the increase in kinetic energy of a mass on which a force is imposed is a function of time.

To summarize, the following steps need to be undertaken to implement an arbitrary passive sampled Port-Hamiltonian system:

- Derive the continuous time Port-Hamiltonian system.
- Perform a causality analysis of each element.
- Define a “change of energy” function for each element. Elements with an admittance causality obtain a time dependent function.
- Define the appropriate energy balances to compute the energy which is stored, dissipated and/or exchanged with other subsystems through a power port.
- Define the hierarchy in which the various energy balances need to be evaluated.
- Monitor the energy flows and implement a strategy to dissipate generated “virtual” energy

V. AN EXAMPLE: PASSIVE INTERACTION WITH A VISCO-ELASTIC MATERIAL

As an example to illustrate the theory explained in the previous section we will consider a single degree of freedom interaction with a piece of virtual material characterized by a SLS model as depicted in Fig. 2. First the various steps of the algorithm with respect to deadlock, energy exchange, and passivity recovery are described, and then simulated and experimental results are presented.

A. Virtual Environment

When the user comes into contact with the virtual wall this is indicated by the condition

$$q(k) < q_w, \quad (24)$$

where $q(k)$ is the measured position of the haptic interface, and q_w is the initial undeformed position of the virtual wall. The user is touching the material with the haptic interface and can therefore only push against the material and not pull on it. This determines the condition at which the contact between the user and the virtual environment is broken as it means that spring S_1 cannot be elongated. When no energy is stored in S_1 , the contact is broken.

1) Deadlock:

The system is initially in a deadlock situation meaning that no energy is stored inside the springs and as such no energy exchange occurs between the user and the virtual environment. When the system is in a deadlock situation, a fraction α of the penetration into the virtual wall is taken as the state change of both the springs and as a result an amount of “virtual” energy ΔH_{dis} is generated (25). This generated energy is to be dissipated by the bookkeeping algorithm of Section V-A.3.

$$\begin{aligned} \Delta x_{S_1}(k) &= \alpha(q_r(k) - q_w) \\ x_{S_1}(\bar{k} + 1) &= \Delta x_{S_1}(k) \\ x_{S_2}(\bar{k} + 1) &= \Delta x_{S_1}(k) \\ \Delta H_{dis}(k) &= \frac{1}{2} k_1 (\Delta x_{S_1}(k))^2 + \frac{1}{2} k_2 (\Delta x_{S_2}(k))^2 \end{aligned} \quad (25)$$

Another deadlock situation arises due to the time dependency of the dissipation in the system as given in (23). Each iteration, energy is dissipated from spring S_2 so the stored energy can be depleted and another deadlock situation for S_2 is obtained. In such a situation the state change $\Delta x_{S_2}(k)$ is

TABLE I
PARAMETER VALUES OF THE SIMULATION AND EXPERIMENT

Parameter	Simulation	Experiment
k_1	500 N/m	50 N/m
k_2	50 N/m	50 N/m
b_1	100 Ns/m	40 Ns/m
q_w	-0.2 m	-0.11 m

chosen equal to $\Delta x_{s_1}(k)$ and the generated “virtual” energy is added to the bookkeeping, similar to (25). The sign of $\Delta x_{s_1}(k)$ determines if S_2 is being compressed or elongated, which defines the direction of the force that S_2 exerts.

2) Energy exchange algorithm:

When energy is being exchanged between the continuous and discrete domains the energy exchange algorithm can be used. The state jump for spring S_1 can be computed using (9), (10), (11) and (12). The state jump for spring S_2 can be computed by combining the computed $H_2(k)$ with (15), (16) and (23). The computation of the discrete state jumps are quadratic problems and have therefore two solutions. The solution with the smallest magnitude is chosen as this results in a new state which has the smallest Euclidian distance to the previous state.

It is possible under certain conditions that there exists no exact solution to the quadratic problem to compute $\Delta x_{s_1}(k)$. Usually this is the case when the user is extracting energy from the virtual environment and a small amount of energy is stored in the system. As discussed in [3] this is a consequence of the ZOH operation. It is possible in these cases to use an approximate solution where the state jump of the previous sample, $\Delta x_{s_1}(k-1)$ is used to determine the energy $\Delta H_2(k)$. (26) shows how to compute $\Delta x_{s_1}(k)$ in such a case.

$$\begin{aligned}
 \Delta H_2(k) &= -F_2 \Delta x_{s_1}(\overline{k-1}) \\
 \Delta H_{s_1}(k) &= \Delta H_I(k) - \Delta H_2(k) \\
 H_{s_1}(\overline{k+1}) &= H_{s_1}(\overline{k}) + \Delta H_{s_1}(k) \\
 x_{s_1}(\overline{k+1}) &= -\sqrt{\frac{2H_{s_1}(\overline{k+1})}{k_1}}
 \end{aligned} \quad (26)$$

This approximate solution given by (26) is used until the energy stored in spring S_1 is depleted or an exact solution to the quadratic problem to compute $\Delta x_{s_1}(k)$ exists.

3) Passivity recovery:

Section V-A.1 discussed where “virtual” energy is generated in the algorithm. The exact amount is recorded by the bookkeeping algorithm and steps need to be undertaken in order to regain passivity according to (4). The same switched damping structure as used in [3] will be applied in this example. This approach is similar to the Passivity Controller part of the algorithm described by Ryu et al. [12]. An additional dissipative force is added to the force computed by the virtual environment to extract additional energy from the user to compensate for the generated “virtual” energy.

B. Results

Both simulations and experiments were carried out for the example of Section V-A using the simulation environment

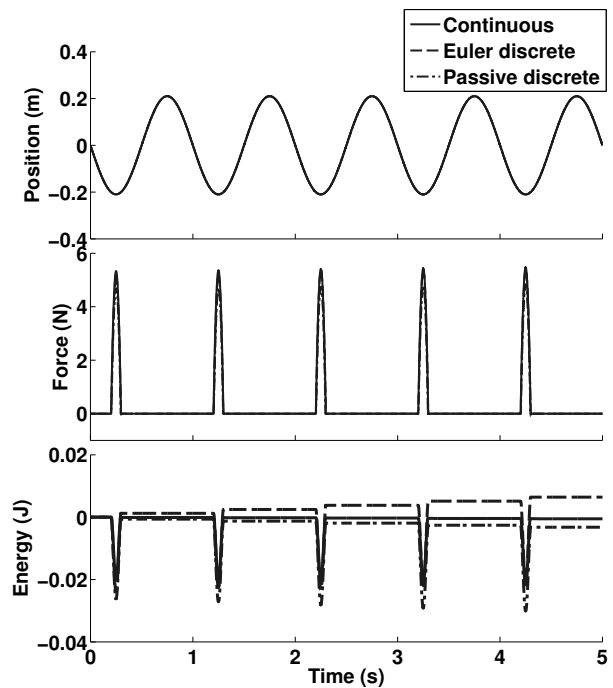


Fig. 5. **Simulation of repetitive contacts:** Plots depict the position, computed force and energy exchanged between the user and the virtual environment.

20-sim [2]. In order to compare the performance also a continuous domain model and a regular Euler discretized model were implemented. In the simulations a sinusoidal motion is applied to each model. The amplitude and frequency of the trajectory are 0.21 m and 0.5 Hz. The values of the parameters of the virtual environment are listed in Table I. These parameters are chosen for illustrative purposes and do not necessarily reflect any physical material. Different parameters for the simulations and experiments are chosen as our experimental setup cannot generate high forces and a virtual environment with a higher stiffness better illustrates the differences between the various models.

Fig. 5 shows the force responses of the three models and the exchanged energy when the discrete models are run at a sample frequency of 1 kHz. It is visible that whereas the continuous domain model and the passively implemented model are both dissipative, the Euler discretized model is non passive. A difference with respect to dissipated energy exists between the continuous and passively implemented model due to the fixed time step dissipation of (23). As the sample period goes to zero both models converge.

In the experiment we show that the passive implementation of the model can display the relaxation effect. The experiment was performed with a Freedom 6S haptic interface [10]. The virtual environment was run in 20sim with a soft real time sample frequency of 100 Hz. The forces that can be rendered by the Freedom 6S and the workspace are limited so the parameters were changed with respect to the simulations as listed in Table I. Fig. 6 shows the relaxation curve of the virtual environment, the force applied to the user as computed by the virtual environment, and the various energy

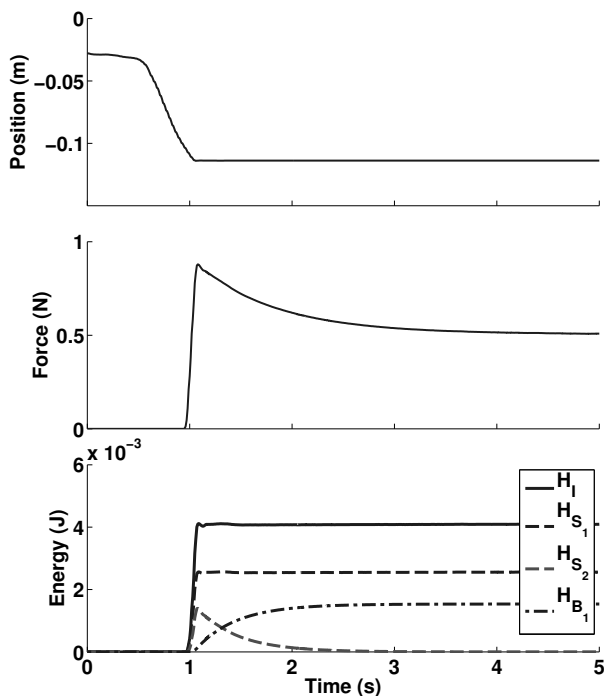


Fig. 6. Experimental data showing the relaxation effect in the virtual environment: H_I is the energy exchanged between the user and the virtual environments, H_{S_1} and H_{S_2} are the energy stored in springs S_1 and S_2 , and H_{B_1} is the accumulated dissipated energy by damper B_1 . The energy stored in spring S_2 (H_{S_2}) is smoothly dissipated by damper B_1 .

functions of the model elements. The energy injected by the user into the virtual environment is given by H_I , the energy stored in springs S_1 and S_2 by H_{S_1} and H_{S_2} , and the accumulated dissipated energy by damper B_1 by H_{B_1} . Initially, both spring S_1 and S_2 are compressed, but the energy stored in S_2 is slowly dissipated by damper B_1 . As a result the force required to sustain this deformation decreases and the relaxation effect is noticeable. Eventually the energy injected by the user equals the energy stored in spring S_1 plus the energy dissipated by damper B_1 , which corresponds to the definition of passivity as given in (4).

VI. CONCLUSIONS AND FUTURE WORK

In this paper we have shown that it is possible to implement sampled multi-dimensional Port-Hamiltonian systems in a passive manner. To this end we needed to introduce a causality analysis of the network and showed that due to the impedance type nature of the virtual environment all network elements with an admittance causality become time dependent. Simulations and experiments with a viscoelastic virtual environment showed that it is possible to implement multi-dimensional virtual environments with internal dynamic behavior with this framework. The framework guarantees the passivity of the virtual environment and also takes the internally dissipated energy into account, whereas a regular discretized model can become non passive.

In the example treated in this paper the multi-dimensional system has been broken down into separate subsystems to compute the energy distribution. Future work will look into the possibility to formulate the computations into a

compact form allowing for a more efficient implementation. Several application areas will be explored using the described framework. A first application of the framework will be the simulation of the haptic interaction with complex virtual environments representative of actual biological tissue. A second application will be in the area of multiple degrees of freedom virtual environments, e.g. the manipulation of a virtual environment with two haptic interfaces. In that scenario, energy is injected into the virtual environment from two interaction points, but the approach to solving the energy balances, according to Section IV is unaltered. The framework can also be applied in telemanipulation schemes. For instance in the algorithm described by Franken et al. [4] where a two layer approach is introduced to separate the passivity and transparency properties of a telemanipulation chain. In [4] the user is interacting with a local model of the remote environment. The result of this paper can be used to implement that local model in a passive manner. In all of these applications benchmark tests will be performed to compare the performance of this framework to that of other existing approaches with respect to transparency, contact stability, and robustness to parameter variations.

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