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OBJECT-ORIENTED MODELING OF THE DYNAMICS OF SPACE SYSTEMS WITH REACTION WHEELS

Sarthak Misra*
Canadian Space Agency - MD Robotics, St-Hubert, Quebec, Canada, J3Y 8Y9.
Arun. K. Misra†
McGill University, Montreal, Quebec, Canada, H3A 2K6

In this paper, an effort has been made to develop objects that can be combined to formulate the complete dynamic equations of a spacecraft containing reaction wheels. In order to generate the mathematical model and dynamical equations of the multibody system, a variation of Lagrange's method has been used, along with the concept of Natural Orthogonal Complement, in order to eliminate the kinematic constraint force and moments. The designed objects would be part of a multibody system software package that could simulate the complex dynamics of a spacecraft containing reaction wheels and any arbitrary configuration of connected bodies. The objects have been designed such that the spin rate of the wheels may be specified as a constant nominal rate, or as any function of time, or in the form of a P.I.D. control law, wherein the wheel spin rate is a function of the body quaternion of the motherbody. The accuracy, versatility, and adaptability of the designed objects have been illustrated with numerous examples and compared with results obtained using standard procedure. Maneuvers have also been simulated on the designed model and compared with available spacecraft data in order to substantiate the authenticity of the designed objects

Introduction

SATELLITES or space systems with connected bodies or appendages belong to the regime of multibody space systems whose dynamics are known to be complex and challenging. The governing equations of motion for the motherbody and the connected bodies can be derived in terms of non-linear differential equations, by modeling them as a multibody mechanical system. The connected bodies may be in the form of solar panels, booms, antennas, and/or manipulator arms. When bodies are attached to the motherbody, there is an interaction between its dynamics and the dynamics of the motherbody. The motion of the connected bodies produces reaction forces and moments on the satellite through the motherbody base. These forces and moments produce translation of the center of mass of the spacecraft and rotation about its center of mass. Due to such dynamic coupling between the motherbody and the connected bodies, the position and the orientation of the spacecraft are functions of the position and orientation of the connected appendages.

Attitude control is one of the most important problems in spacecraft design, since there are always disturbances due to dynamic interaction, gravitational torques, orbital motion, etc.[1] The spacecraft attitude stabilization is necessary in order to maintain communication links, generate electrical power from the solar panels, and to comply with the mission objectives of the spacecraft. The dynamic coupling between the attached appendages and the spacecraft poses control problems. Reaction wheels are used in many space applications in order to maintain and/or reorient the attitude of the spacecraft. The wheels are located at a fixed orientation with respect to the spacecraft body axes. The motion of the attached appendages of a multibody space system causes variation in the attitude of the spacecraft, which in turn causes the angular momentum of the spacecraft to change. The spacecraft attitude is controlled by absorbing the angular impulses from the external torques into the reaction wheels during slew or reorientation maneuvers. This transferral is accomplished by applying control laws to the wheels. These wheels are aptly called reaction wheels because the equal and opposite torque from the wheels on the spacecraft tend to cancel the external torque, leaving the momentum of the spacecraft unchanged.[2]

Considerable effort has been directed towards

*Engineer, MSS Mission Operations and Analysis.
†Professor, Dept. of Mechanical Engineering, AIAA member.

the dynamics, modeling, and computer simulation of multibody systems because the micro-gravity environment of space is not easily amenable for experimentation on the ground.[3] Most previous investigations have considered specific models and have adopted a procedure-oriented approach. In this method of computation, the main task is broken down into numerous simple instruction units that are processed in a serial fashion. This method is prone to errors and does not easily accommodate the possibility of modifications, as it is too laborious. The object-oriented modeling consists of identifying objects and the computations done by those objects, and creating simulations of those objects, their processes, and the required communications between the objects.[4] Such a formulation technique is less prone to errors and is relatively easy to modify and/or upgrade.

Due to the apparent and numerous advantages of using object-oriented modeling techniques, various researchers have applied these concepts for the dynamic simulation of multibody systems.[3,5–9] Otter and Hocke can be credited as one of the first research groups who introduced the concept of object-oriented programming into a data model for the exchange of rigid multibody system descriptions.[6] Wallrapp extended the work of Otter and Hocke by adding classes that could represent the flexible members of the multibody system.[7] Kecskeméthy applied object-oriented modeling for the efficient generation and solution of the equations of motion for rigid multibody systems.[8] Lückel et al. adopted a computationally efficient recursive Newton-Euler formalism for the dynamic simulation of rigid multibody systems.[9] The use of work-energy relations was the focus of research for the dynamic simulation of multibody systems by Anantharaman.[10]

In order to accommodate the flexibility as well as maintain the independent representation of each individual body in a multibody system, Min et al. adopted the formalism based on the Euler-Lagrange method in conjunction with the use of the Natural Orthogonal Complement of the kinematic constraint matrix.[3] In that work, the multibody dynamics model has been treated as a combination of **Body**, **Joint**, **System**, and **Solver** classes. The objects defined in these classes are refined from the generic types in a top-down fashion by taking advantage of the inheritance concept, as opposed to the procedure-oriented approach which functions at a very specific level. The methodology followed in this research is an extension of the work done by Min et al. This paper involves the modeling of objects that would simulate the dynamic response of a spacecraft carrying reaction wheels. The spacecraft could be a complex multibody space system.

Dynamical Equations of the System

The system under study is composed of a main body (motherbody or satellite) containing reaction wheels, that serves as a platform on which there is provision for multiple appendages to be attached in any open chain configuration. The main body or satellite is modeled to be a rigid body, while the multiple short appendages are modeled to be rigid links. The joints are modeled as rigid and could be of prismatic, revolute or free floating type.

The derivations of the dynamical equations of the spacecraft with reaction wheels and any other connected bodies has been done using a variation of the Euler-Lagrange formulation method. In the case of this variation of Lagrangian dynamics, the kinetic and potential energies of each body are considered and then the equations of motion are derived. The equations of motion of each body are then assembled to get the dynamical equations motion of the whole system. This method introduces the non-working constraint wrenches, as in the case of the Newton-Euler formulation, but they are eliminated using the concept of Natural Orthogonal Complement.[11]

Formulation of Equations of Motion for the i^{th} Body

The kinetic energy for each body, which could be the spacecraft, motherbody, or any attached appendage is,

$$T_i = T_i\,(\mathbf{q}_i\,, \dot{\mathbf{q}}_i) \tag{1}$$

, where in the 7−dimensional extended position vector, $\mathbf{q}_i$, contains the position vector $\mathbf{p}_i$ locating any point on the i^{th} body and the orientation $\hat{\mathbf{q}}_i$ of that body. It is also referred to as the pose of the body. The 6−dimensional extended velocity vector $\mathbf{v}_i$ consists of the linear or translational velocity $\dot{\mathbf{p}}_i$ of any point on the i^{th} body and the angular velocity $\boldsymbol{\omega}_i$ of that body. It is also referred to as the twist. The twist $\mathbf{v}_i$ and the time derivative of the pose $\dot{\mathbf{q}}_i$ are related as,

$$\mathbf{v}_i = \mathbf{L}_i\,\dot{\mathbf{q}}_i \tag{2}$$

where $\dot{\mathbf{q}}_i$ is a 7×1 vector, and $\mathbf{L}_i = \mathbf{L}_i\,(\mathbf{q}_i,\, t)$ which is a 6×7 matrix. Also,

$$\dot{\mathbf{q}}_i = \boldsymbol{\Lambda}_i\,\mathbf{v}_i \tag{3}$$

where $\boldsymbol{\Lambda}_i = \boldsymbol{\Lambda}_i\,(\mathbf{q}_i,\, t)$ is a 7×6 matrix. The kinetic energy can be written in terms of the twist $\mathbf{v}_i$ as,

$$T_i = \frac{1}{2}\,\mathbf{v}_i^T\,\mathbf{M}_i\,\mathbf{v}_i \tag{4}$$

where $\mathbf{M}_i = M_i(\mathbf{q}_i, t)$ is the 6×6 extended mass matrix of body i. The expression for the extended mass matrix can be derived from the kinetic energy and is written as,

$$\mathbf{M}_i = \begin{bmatrix} \mathbf{M}_i^{dd} & \mathbf{M}_i^{dr} \\ \mathbf{M}_i^{rd} & \mathbf{M}_i^{rr} \end{bmatrix} \tag{5}$$

where,

$$\mathbf{M}_i^{dd} \equiv \int_i \mathbf{1}\, dm_i = m_i\, \mathbf{1} \tag{6}$$

$$\mathbf{M}_i^{dr} \equiv -\int_i \mathbf{r}_{o_i}^{\times}\, dm_i = -m_i\, \mathbf{c}_{o_i}^{\times} \tag{7}$$

$$\mathbf{M}_i^{rr} \equiv -\int_i \mathbf{r}_{o_i}^{\times}\, \mathbf{r}_{o_i}^{\times}\, dm = \mathbf{M}_{o_i}^{rr} \tag{8}$$

In the above equations m_i, $\mathbf{c}_{o_i}$ and $\mathbf{M}_{o_i}^{rr}$ are the total mass, the position vector of the center of mass, and the second moment of inertia of the i^{th} body around its center of mass, respectively.
The dynamical equations of body i are derived using the Euler-Lagrange equation, which is given as,

$$\frac{d}{dt}\left(\frac{\partial T_i}{\partial \dot{\mathbf{q}}_i}\right) - \frac{\partial T_i}{\partial \mathbf{q}_i} = \mathbf{w}_i \tag{9}$$

where, $\mathbf{w}_i$ is a 6×1 dimensional vector accounting for all nonconservative wrenches.

$$\mathbf{w}_i = \mathbf{w}_i^E + \mathbf{w}_i^K + \mathbf{w}_i^A \tag{10}$$

where,

- $\mathbf{w}_i^A$ is the algebraic constraint wrench originating from the fact that,

$$\hat{\mathbf{q}}_i^T\, \hat{\mathbf{q}}_i = \mathbf{1} \tag{11}$$

- $\mathbf{w}_i^E$ is the external wrench.
- $\mathbf{w}_i^K$ is the kinematic constraint wrench.

The potential energy has been ignored in the Euler-Lagrange equation (9). Such a consideration has been done because of the fact that in a micro-gravity environment the effect of the gravitational potential energy is small compared to the nonconservative wrenches. The equation of motion for the i^{th} body, using the relations derived by Misra et al. can be expressed as,[12]

$$\begin{aligned} \mathbf{M}_i\, \dot{\mathbf{v}}_i = & -\dot{\mathbf{M}}_i\, \mathbf{v}_i - 2\, \boldsymbol{\Lambda}_i^T\, \dot{\mathbf{L}}_i^T\, \mathbf{M}_i\, \mathbf{v}_i \\ & + \frac{1}{2}\, \boldsymbol{\Lambda}_i^T \left(\mathbf{v}_i^T\, \frac{\partial \mathbf{M}_i}{\partial \mathbf{q}_i}\, \mathbf{v}_i\right) \\ & + \boldsymbol{\phi}_i^E + \boldsymbol{\phi}_i^K \end{aligned} \tag{12}$$

where,

$$\boldsymbol{\phi}_i^E = \boldsymbol{\Lambda}_i^T\, \mathbf{w}_i^E$$

$$\boldsymbol{\phi}_i^K = \boldsymbol{\Lambda}_i^T\, \mathbf{w}_i^K$$

Equation (12) can be stated in a compact form as,

$$\mathbf{M}_i\, \dot{\mathbf{v}}_i = \boldsymbol{\phi}_i^S + \boldsymbol{\phi}_i^E + \boldsymbol{\phi}_i^K \tag{13}$$

which is the generalized Newton-Euler equation, where $\boldsymbol{\phi}_i^S$ is the system wrench given by,

$$\boldsymbol{\phi}_i^S = -\dot{\mathbf{M}}_i\, \mathbf{v}_i - 2\, \boldsymbol{\Lambda}_i^T\, \dot{\mathbf{L}}_i^T\, \mathbf{M}_i\, \mathbf{v}_i + \frac{1}{2}\, \boldsymbol{\Lambda}_i^T \left(\mathbf{v}_i^T\, \frac{\partial \mathbf{M}_i}{\partial \mathbf{q}_i}\, \mathbf{v}_i\right) \tag{14}$$

Dynamical Equations of Motion for a Spacecraft with Reaction Wheels

The spacecraft can be regarded as a rigid body and hence the concepts and formulation techniques that were introduced in the previous section for a body i could be extended to the spacecraft (subscript B). The reaction wheels are mounted on a fixed axis frame with respect to the body coordinate frame of the spacecraft. The kinetic energy of a spacecraft containing a reaction wheel is given by,

$$T_{SC} = T_B + T_w \tag{15}$$

where,

- T_{SC} is the total kinetic energy
- T_B is the kinetic energy of the spacecraft
- T_w is the kinetic energy of the reaction wheel

Thus the total kinetic energy T_{SC} can be separated into two parts T_B and T_w. In the foregoing formulation the Euler-Lagrange equation is applied to the sum of T_B and T_w. However, for the ease of formulation the contributions from T_B and T_w are evaluated separately and then the evaluated terms are added together in order to get the complete dynamic equations of motion of the spacecraft with the reaction wheels.

The Spacecraft

The kinetic energy (T_B) for the spacecraft is given as,

$$T_B = \frac{1}{2}\, \mathbf{v}_B^T\, \mathbf{M}_B\, \mathbf{v}_B \tag{16}$$

where,

- $\mathbf{v}_B$ is the 6−dimensional twist of the spacecraft.
- $\mathbf{M}_B$ is the 6×6 extended mass matrix of the spacecraft.

On applying the Euler-Lagrange equation to equation (16) and using the result obtained from equation (12), the contribution of the kinetic energy of the motherbody to the equation of motion of the considered system is,

$$\begin{aligned}\mathbf{M}_B\,\dot{\mathbf{v}}_B \;=\; & -\dot{\mathbf{M}}_B\,\mathbf{v}_B \,-\, 2\,\mathbf{\Lambda}_B^T\,\dot{\mathbf{L}}_B^T\,\mathbf{M}_B\,\mathbf{v}_B \\ & + \frac{1}{2}\,\mathbf{\Lambda}_B^T\left(\mathbf{v}_B^T\,\frac{\partial \mathbf{M}_B}{\partial \mathbf{q}_B}\,\mathbf{v}_B\right) \\ & + \boldsymbol{\phi}_B^E \,+\, \boldsymbol{\phi}_B^K \end{aligned} \tag{17}$$

Reaction Wheel

The derivation presented below is for one reaction wheel which can be extended to three reaction wheels. The kinetic energy of the wheel is given as,

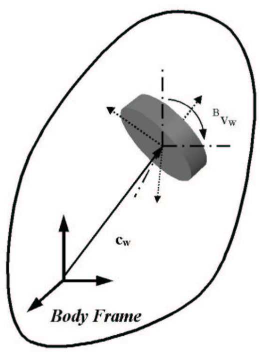

Fig. 1 Reaction wheel in the motherbody

$$T_w \,=\, \frac{1}{2}\,\mathbf{v}_w^T\,\mathbf{M}_w\,\mathbf{v}_w \tag{18}$$

where $\mathbf{M}_w$ is the 6×6 extended mass matrix of reaction wheel. $\mathbf{c}_w$ is the position vector of the center of mass of the wheel from the origin of the body frame of the motherbody, as shown in figure 1. ${}^B\mathbf{v}_w$ is the twist of the wheel relative to the motherbody. The total twist of the reaction wheel, $\mathbf{v}_w$, which is expressed in the body frame of the motherbody is given by,

$$\mathbf{v}_w \,=\, \mathbf{v}_B \,+\, {}^B\mathbf{v}_w \tag{19}$$

${}^B\mathbf{v}_w$ is a 6-dimensional vector. All the elements of ${}^B\mathbf{v}_w$ are zero, except one, depending on the axis of spin of the reaction wheel. The fourth, fifth or sixth element of the vector ${}^B\mathbf{v}_w$ constitutes the spin rate (relative angular velocity) of wheel if it spins about its X-axis, Y-axis or Z-axis, respectively. This relative angular speed of the reaction wheel is denoted by ω_{wheel} and it could be:

1. A constant nominal spin rate.
2. Any function of time.
3. In the form of a P.I.D. control law.

The inertia (Left hand) terms and the wrench (Right hand) terms due to the kinetic energy of the reaction wheel derived by Misra et al. are combined with the inertia and wrench terms of the spacecraft to get the dynamical equations of motion for the spacecraft containing the reaction wheel.[12] The inertia and wrench terms of the spacecraft have been derived in equation (17). Thus, the complete equation of motion of the spacecraft with the reaction wheel is,

$$\begin{aligned}\mathbf{M}_B\,\dot{\mathbf{v}}_B \quad & + \mathbf{M}_w\,\dot{\mathbf{v}}_B \,+\, \mathbf{\Lambda}_B^T\left(\frac{\partial\, {}^B\mathbf{v}_w}{\partial \dot{\mathbf{q}}_B}\right)^T \mathbf{M}_w\,\dot{\mathbf{v}}_B \;= \\ & -\dot{\mathbf{M}}_B\,\mathbf{v}_B \,-\, 2\,\mathbf{\Lambda}_B^T\,\dot{\mathbf{L}}_B^T\,\mathbf{M}_B\,\mathbf{v}_B \\ & + \frac{1}{2}\,\mathbf{\Lambda}_B^T\left(\mathbf{v}_B^T\,\frac{\partial \mathbf{M}_B}{\partial \mathbf{q}_B}\,\mathbf{v}_B\right) \\ & - 2\,\mathbf{\Lambda}_B^T\,\dot{\mathbf{L}}_B^T\,\mathbf{M}_w\,\mathbf{v}_B \,+\, \mathbf{\Lambda}_B^T\left(\frac{\partial\, {}^B\mathbf{v}_w}{\partial \mathbf{q}_B}\right)^T \mathbf{M}_w\,\mathbf{v}_B \\ & - 2\,\mathbf{\Lambda}_B^T\,\dot{\mathbf{L}}_B^T\,\mathbf{M}_w\,{}^B\mathbf{v}_w \,-\, \mathbf{M}_w\,\frac{d\,{}^B\mathbf{v}_w}{dt} \\ & - \mathbf{\Lambda}_B^T\,\frac{d}{dt}\left(\frac{\partial\, {}^B\mathbf{v}_w}{\partial \dot{\mathbf{q}}_B}\right)^T \mathbf{M}_w\mathbf{v}_B \\ & - \mathbf{\Lambda}_B^T\left[\frac{d}{dt}\left(\frac{\partial\, {}^B\mathbf{v}_w}{\partial \dot{\mathbf{q}}_B}\right)^T \mathbf{M}_w\,{}^B\mathbf{v}_w\right] \\ & - \mathbf{\Lambda}_B^T\left[\left(\frac{\partial\, {}^B\mathbf{v}_w}{\partial \dot{\mathbf{q}}_B}\right)^T \mathbf{M}_w\,\frac{d\,{}^B\mathbf{v}_w}{dt}\right] \\ & + \mathbf{\Lambda}_B^T\left[\left(\frac{\partial\, {}^B\mathbf{v}_w}{\partial \mathbf{q}_B}\right)^T \mathbf{M}_w\,{}^B\mathbf{v}_w\right] \\ & + \boldsymbol{\phi}_B^E \,+\, \boldsymbol{\phi}_B^K \end{aligned} \tag{20}$$

The equation of motion for any i^{th} connected body appended onto the spacecraft is given in equation (12). Equation (20) is the dynamic equation of motion of the motherbody containing a reaction wheel. The inertia and wrench terms of this equation have been converted into programmable code and incoporated into functional objects.

Object-oriented Concepts

Kinematic Objects

As mentioned previously, multibody space systems can be composed of many distinctive components, such as solar panels, booms, manipulator links, antennas, actuators, reaction wheels, control moment gyros, and jet thrusters. The processes and operations for each of these components can be identified and characterized into objects. The following section describes the essential kinematic relationships that have been embedded into objects for the transfer of motion states from one body to the next body via joints in a multibody system. The objects mentioned in this paper are written in *italics.*

The Generic Element

Kinematics is responsible for holding and passing the motion states of the body. The transfer of motion states can occur within a *Link* and within a *Joint* due to the motion of the rigid body. The relationships presented below are given in the most general form for a generic kinematic element, during the transfer of motion states from one frame to another. These relationships have been extended to be incorporated in the objects *Link* and *Joint.*

The fundamental motion states of the rigid body are position ($\mathbf{p}$), velocity ($\dot{\mathbf{p}}$) and acceleration ($\ddot{\mathbf{p}}_{\ddot{\mathbf{y}}=0}$) for translation, and rotation matrix ($\mathbf{R}$), angular velocity ($\boldsymbol{\omega}$), and angular acceleration ($\dot{\boldsymbol{\omega}}_{\ddot{\mathbf{y}}=0}$) for rotation respectively. They are held in an object titled *Frame.* In the above $(\cdot)_{\ddot{\mathbf{y}}=0}$ stands for the evaluation of a quantity when the generalized acceleration vector ($\ddot{\mathbf{y}}$) of the multibody system is set to zero, where $\mathbf{y}$ is the generalized coordinate vector of the system. These values are used instead of real accelerations, because they are useful for the assembly of the global system dynamics. The kinematic motion states are expressed with respect to the inertial reference frame $\mathbf{O}$. The frame $\mathbf{O}_m$ represents the local body frame of the rigid body. $\mathbf{O}_s$ represents the new frame after the relative change in position and orientation due to rigid body motion. $\mathbf{p}_s$ is the position vector that locates the origin of the frame $\mathbf{O}_s$ from the origin of the inertial reference frame $\mathbf{O}$, while $\mathbf{R}_s$ is the rotation matrix that represents the orientation of the frame $\mathbf{O}_s$ with respect to the inertial reference frame $\mathbf{O}$. $\mathbf{p}_m$ is the position vector that locates the origin of the frame $\mathbf{O}_m$ from the origin of the inertial reference frame $\mathbf{O}$, while $\mathbf{R}_m$ is the rotation matrix that represents the orientation of the frame $\mathbf{O}_m$ with respect to the inertial reference frame $\mathbf{O}$. The fundamental operation of the generic kinematic element is the transmission of motion states from one frame into another as depicted in figure 2. It consists of the following three transmissions.

- Displacement

$$\mathbf{R}^T\,(\mathbf{p}_m + \mathbf{r}) = \mathbf{p}_s \tag{21}$$

$$\mathbf{R}_m\,\mathbf{R} = \mathbf{R}_s \tag{22}$$

- Velocity

$$\mathbf{R}^T\,\left(\dot{\mathbf{p}}_m + \boldsymbol{\omega}_m^{\times}\,\mathbf{r}\right) = \dot{\mathbf{p}}_s \tag{23}$$

$$\mathbf{R}^T\,(\boldsymbol{\omega}_m + \boldsymbol{\omega}) = \boldsymbol{\omega}_s \tag{24}$$

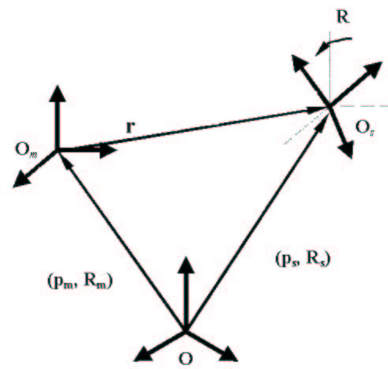

Fig. 2 Generic kinematic element

- Acceleration

$$\mathbf{R}^T\,\left(\ddot{\mathbf{p}}_{m_{\ddot{\mathbf{y}}=0}} + \dot{\boldsymbol{\omega}}^{\times}_{m_{\ddot{\mathbf{y}}=0}}\,\mathbf{r} + \boldsymbol{\omega}_m^{\times}\,\boldsymbol{\omega}_m^{\times}\,\mathbf{r}\right) = \ddot{\mathbf{p}}_{s_{\ddot{\mathbf{y}}=0}} \tag{25}$$

$$\mathbf{R}^T\,\left(\dot{\boldsymbol{\omega}}_{m_{\ddot{\mathbf{y}}=0}} + \boldsymbol{\omega}_m^{\times}\,\boldsymbol{\omega}\right) = \dot{\boldsymbol{\omega}}_{s_{\ddot{\mathbf{y}}=0}} \tag{26}$$

In the above expressions, $\mathbf{r}$ and $\boldsymbol{\omega}$ represent the relative displacement and relative angular velocity, respectively, between frames $\mathbf{O}_m$ and $\mathbf{O}_s$, while the superscript '$\times$' represents the cross product operation.

Link

Link is a kinematic object that transfers motion states within a body. In order to provide some simplicity while modeling complex multibody systems, the objects *Outward Link* and *Inward Link* are developed for a rigid body. The *Outward Link* is responsible for transfer of kinematic states from the local body frame $\mathbf{O}_B$ to the outside frame $\mathbf{O}_J$ (the term "outside frame" represents the frame towards the terminal bodies/outside of the multibody system, at the connection point between the joint and the body). The *Inward Link* transfers the motion states from the inside frame $\mathbf{O}_J$ (the term "inside frame" represents the frame towards the base/inside of the multibody system, at the connection point between the joint and the body) to the local body frame $\mathbf{O}_B$. The figure 3 depicts the schematic of a generic *Link* (*Outward Link* and *Inward Link*). $\mathbf{r}$ is the nominal length vector from the origin of frame $\mathbf{O}_B$ to the origin of the frame $\mathbf{O}_J$, while $\mathbf{R}$ is the rotation matrix of frame $\mathbf{O}_J$ with respect to frame $\mathbf{O}_B$ due to normal twist of the body. $\mathbf{r}$ is a constant vector and $\mathbf{R}$ is a constant matrix and they are determined by the nominal body architecture.

Along with the transmission of motion states, the link restrains the relative motion of a frame with respect to another. The kinematic constraint equations of the link can be obtained by the modification of the velocity transmission relations, equations (23) and

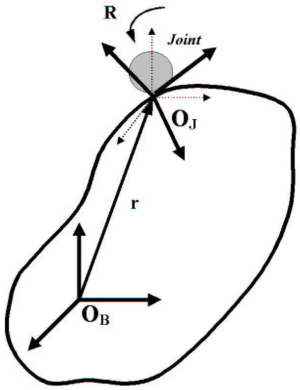

Fig. 3 Generic rigid link

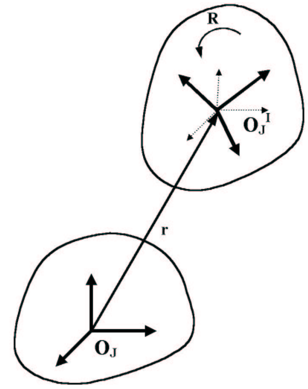

Fig. 4 Generic joint

(24). These constraint equations have been derived by Min et al. and Misra et al.[3,12,13]

Joint

Joint is an object that transfers the kinematic states between two interconnecting bodies. Joints can be of revolute, prismatic, universal, or spherical types. Figure 4 depicts the schematic of a generic *Joint.* $\mathbf{O}_J$ and $\mathbf{O}_J^I$ represent the joint frames before and after the joint has undergone motion (translation and/or rotation), respectively. If $\mathbf{r}$ is a vector that represents the translational displacement of the joint, as in the case of a prismatic joint, and if $\mathbf{R}$ is a matrix that represents the rotation of the joint, as in the case of a revolute joint, then the state quantities of the *Joint* are expressed below in table 1 as,

In table 1, $\boldsymbol{\theta}$ is the joint coordinate vector composed of the translational displacement and/or rotational angles of the joint. $\mathbf{Z}_v$, $\dot{\mathbf{Z}}_v$, $\mathbf{Z}_\omega$, and $\dot{\mathbf{Z}}_\omega$ are the joint characteristic matrices and are dependent on the type of joint. For example, $(\mathbf{Z}_v, \dot{\mathbf{Z}}_v, \mathbf{Z}_\omega, \dot{\mathbf{Z}}_\omega) = (\hat{\mathbf{z}}_p, \mathbf{0}_{3\times1}, \mathbf{0}_{3\times1}, \mathbf{0}_{3\times1})$ for a prismatic joint undergoing translation along the $\hat{\mathbf{z}}_p$ axis, while $(\mathbf{Z}_v, \dot{\mathbf{Z}}_v, \mathbf{Z}_\omega, \dot{\mathbf{Z}}_\omega) = (\mathbf{0}_{3\times1}, \mathbf{0}_{3\times1}, \hat{\mathbf{z}}_r, \mathbf{0}_{3\times1})$ for a revolute joint undergoing rotation about the $\hat{\mathbf{z}}_r$ axis.

Translation	Rotation
$\mathbf{r} = r(\boldsymbol{\theta})$	$\mathbf{R} = R(\boldsymbol{\theta})$
$\dot{\mathbf{r}} = \mathbf{Z}_v \dot{\boldsymbol{\theta}}$	$\boldsymbol{\omega} = \mathbf{Z}_\omega \dot{\boldsymbol{\theta}}$
$\ddot{\mathbf{r}}_{\ddot{\mathbf{y}}=0} = \dot{\mathbf{Z}}_v \dot{\boldsymbol{\theta}} + \mathbf{Z}_v \ddot{\boldsymbol{\theta}}_{\ddot{\mathbf{y}}=0}$	$\dot{\boldsymbol{\omega}}_{\ddot{\mathbf{y}}=0} = \dot{\mathbf{Z}}_\omega \dot{\boldsymbol{\theta}} + \mathbf{Z}_\omega \ddot{\boldsymbol{\theta}}_{\ddot{\mathbf{y}}=0}$

Table 1 State quantities of a joint

A joint also defines a restriction on the relative motion of a frame on one body with respect to a frame on an adjacent body. As mentioned previously, from the velocity transmission relations given by equations (23) and (24), the kinematic constraint equation for the joint has been derived by Min et al. and Misra et al.[3,12,13]

Dynamic Objects

Dynamic objects are responsible for yielding the equation of motion of a body, which is given in the general form as,[14]

$$\mathbf{M}(\mathbf{y})\,\dot{\mathbf{v}} = \boldsymbol{\phi}(\mathbf{y}, \mathbf{v}, t) \tag{27}$$

where,

- $\mathbf{M}$ is the 6×6 extended mass matrix of the body.
- $\boldsymbol{\phi}$ is the 6−dimensional wrench of the body consisting of the external forces and moments acting on the body, and the mixed terms i.e. terms which are functions of the generalized coordinate vector $\mathbf{y}$, extended velocity vector $\mathbf{v}$ and also time t.

The generic dynamics objects are *Inertia* and *Wrench*, which represents the motion resistance and the motion agent, respectively. The primary operation of *Inertia* is to build the extended mass matrix and its time derivative of the body, while *Wrench* is responsible for evaluation of the right hand side of equation (27).

Assembly

Multibody systems in the most general form can be considered to be composed of N bodies. After assembling the individual body equations, which includes the motherbody and the connected auxiliary bodies, the dynamical equations of the complete multibody system are obtained in the form[11,15]

$$\mathbf{M}\dot{\mathbf{v}} = \boldsymbol{\phi}^S + \boldsymbol{\phi}^E + \boldsymbol{\phi}^K \tag{28}$$

where $\mathbf{M}$ is the $6N \times 6N$ generalized mass matrix, $\dot{\mathbf{v}}$ represents the $6N$−dimensional time derivative of the generalized twist, while $\boldsymbol{\phi}^S$, $\boldsymbol{\phi}^E$, and $\boldsymbol{\phi}^K$ are the $6N$−dimensional generalized system, generalized external, and generalized constraint wrenches, respectively.

The dynamical equations of a multibody system written in equation (28) contain non-working constraint wrenches due to the physical coupling between adjacent bodies. These constraint wrenches introduce additional variables in the dynamical equations, as a result, the dimension of the system of equations is increased. Thus the constraint equations are not desirable and hence, have to be eliminated from the dynamical equations. The Natural Orthogonal Complement matrix $\mathbf{N}$ maps the generalized speed $\dot{\mathbf{y}}$ and the generalized twist $\mathbf{v}$, such that,

$$\mathbf{v} = \mathbf{N}\,\dot{\mathbf{y}} \tag{29}$$

The Natural Orthogonal Complement matrix is derived from the kinematic constraint equations of the link and the joint.[3,12] It has been shown that, by using the principle of virtual work and the fact that the kinematic constraint wrench is a non-working wrench, the Natural Orthogonal Complement matrix $\mathbf{N}$ is orthogonal to the kinematic constraint wrench $\boldsymbol{\phi}^K$.[11] Thus,

$$\mathbf{N}^T\,\boldsymbol{\phi}^K = \mathbf{0} \tag{30}$$

Taking the time derivative of the generalized twist $\mathbf{v}$, given in equation (29),

$$\dot{\mathbf{v}} = \dot{\mathbf{N}}\,\dot{\mathbf{y}} + \mathbf{N}\,\ddot{\mathbf{y}} \tag{31}$$

Pre-multiplying the dynamical equation of motion (28) of the multibody system with $\mathbf{N}^T$, and using the relations derived in equations (29), (30), and (31), the dynamical equation of motion of the multibody system is,

$$\left(\mathbf{N}^T\mathbf{M}\mathbf{N}\right)\,\ddot{\mathbf{y}} = -\mathbf{N}^T\mathbf{M}\,\dot{\mathbf{N}}\,\dot{\mathbf{y}} + \mathbf{N}^T\,\boldsymbol{\phi}^S + \mathbf{N}^T\,\boldsymbol{\phi}^E \tag{32}$$

Evaluation of $\dot{\mathbf{N}}^T$ involves extensive computations, hence the following relation is used,

$$\dot{\mathbf{N}}\,\dot{\mathbf{y}} = \dot{\mathbf{v}}_{\ddot{\mathbf{y}}=0} \tag{33}$$

Using equation (33), equation (32) is rewritten in a more compact form. This is the equation of motion of the complete multibody system obtained in terms of the minimal generalized coordinates as,

$$\mathbf{I}\,\ddot{\mathbf{y}} = \mathbf{c}\,(\mathbf{y}, \dot{\mathbf{y}}) + \boldsymbol{\tau} \tag{34}$$

where,

$$\mathbf{I} = \mathbf{N}^T\mathbf{M}\mathbf{N}$$
$$\mathbf{c} = \mathbf{N}^T\left(-\mathbf{M}\,\dot{\mathbf{v}}_{\ddot{\mathbf{y}}=0} + \boldsymbol{\phi}^S\right)$$
$$\boldsymbol{\tau} = \mathbf{N}^T\,\boldsymbol{\phi}^E$$

Implementation

The multibody system under consideration has a base or a motherbody to which a series of bodies can be attached in any open chain configuration. The appendages are connected to each other via joints. The kinematics and dynamics of these various elements of the multibody space system have been formulated and coded into objects. Objects having common functionalities and characteristics can be grouped into a class. In this research work, the multibody simulation archetype constitutes of **Body**, **Joint**, **System**, and **Solver** classes.[13]

Body

The class **Body** is an abstraction of objects having mass as well as finite dimensions. The objects grouped in this class represent the functionality that characterizes the complete kinematics and dynamics of a rigid body. The objects *Frame*, *Base*, *Link*, *Inertia*, and *Wrench* belong to the class **Body**.

The object *Frame* characterizes the local body frame of the rigid body. It stores the kinematic states of the body and is updated recursively from the base to the terminal bodies of the system. The kinematic states of the body include the position and orientation, the linear and angular velocities, and the linear and angular accelerations of the body. The transmission of these kinematic states from one frame to the next frame is through the objects like *Link* and *Joint*.

A multibody system has an inertial reference frame. The local body frame of each body in the multibody system has been resolved in the inertial reference frame, in order to enable the global dynamics assembly of the system. The *Base* is reckoned as a special type of the frame having a stationary kinematic state. This implies that the coordinate frame associated with the *Base* is regarded as the inertial reference frame of the system. It is possible to induce motion to the base or motherbody in the form of external velocity and acceleration. This enables the simulation of a gravity environment by providing an external linear acceleration to the *Base*. Orbital motion of the spacecraft can also be simulated by providing external angular velocity to the *Base*.

In a complex multibody system each body can be connected with one or more bodies. Hence the objects must be designed so as to accommodate any such general formulation. A body is separated into two parts and is visualized as two distinguishable objects, viz. *Inward Link* and *Outward Link*. As shown in figure

5, the *Outward Link* of body i is connected to *Inward Links* of bodies $i+1$ and $i+2$, which characterizes the fact that bodies *i+1* and *i+2* are both connected to body *i*. The objects *Inward Link* and *Outward Link* are designed to transfer kinematic states within a body. The spatial transformation matrices derived,[3,12] are computed in the objects *Outward Link* and *Inward Link*, respectively, which are used for the formulation of the Natural Orthogonal Complement matrix in the *System* object.

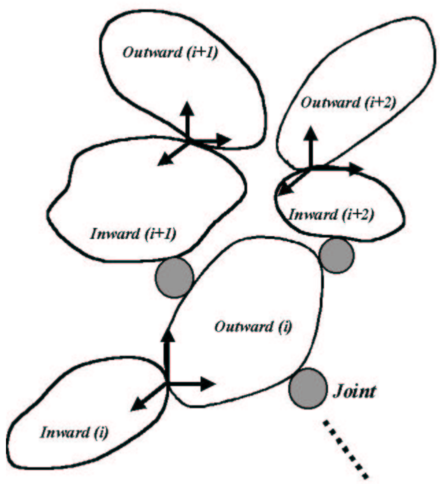

Fig. 5 Abstraction of Inward and Outward Links

Inertia and *Wrench* are built for the dynamics of the system. The dynamic equation of motion for any body (motherbody or any connected appendage), in terms of the extended velocity vector $\mathbf{v}_i$, is given by,[11]

$$\mathbf{M}_i\,\dot{\mathbf{v}}_i = \boldsymbol{\phi}_i^S + \boldsymbol{\phi}_i^E + \boldsymbol{\phi}_i^K \tag{35}$$

where $\mathbf{M}_i$ is the extended mass matrix of the body, while $\boldsymbol{\phi}_i^S$, $\boldsymbol{\phi}_i^E$ and $\boldsymbol{\phi}_i^K$ are the wrenches due to the non-inertial and relative motions of the body, the external forces and moments, and the kinematic constraints imposed by the links and joints connected to the body, respectively.

The object *Inertia* computes the extended mass matrix and its time derivative of any body attached to the motherbody. The object *Inertia (Body and Wheel)* computes the motherbody's extended mass matrix and its time derivative, and also the extended mass matrix of the reaction wheels. The motherbody's extended mass matrix and its time derivative are computed if the mass, mass moment of inertia, and location of the center of mass of the motherbody with respect to the local body coordinate frame are supplied to the object. The extended mass matrix of the reaction wheel is also computed once its mass and mass moment of inertia are inputed to the object *Inertia (Body and Wheel).*

The dynamic object *Wrench* computes the wrench of any body connected to the motherbody. It combines all the motion agents on the body and evaluates the right hand side of equation (35). During the global dynamics assembly of the system, another term having dimension of wrench appears and it is accounted for in the system wrench of the individual body. This term appears due to the use of the expression $\dot{\mathbf{N}}\dot{\mathbf{y}} = \dot{\mathbf{v}}_{\ddot{\mathbf{y}}=0}$, as derived in equation (33). Therefore, the system wrench of the body which is computed in this object is given by,

$$\boldsymbol{\phi}_i = -\mathbf{M}_i\,\dot{\mathbf{v}}_{i_{\ddot{\mathbf{y}}=0}} + \boldsymbol{\phi}_i^S \tag{36}$$

The noteworthy attributes of the object *Wrench (Body and Wheel)* are:

1. It is feasible to place the reaction wheels in any orientation with respect to the body coordinate frame.
2. There is a capability to specify a constant nominal spin rate of the reaction wheel.
3. One can specify the angular velocity of the reaction wheel as any function of time.
4. It is possible to specify the angular velocity of the wheel as a function of the body quaternion i.e. the wheel velocity could be in the form of a P.I.D. control law. Thus it is possible to specify the spin rate of each wheel in the form,

$$\omega_{wheel} = \mathbf{K}_1\,\hat{\mathbf{q}}_B + \mathbf{K}_2\int\hat{\mathbf{q}}_B\,dt + \mathbf{K}_3\,\dot{\hat{q}}_B \tag{37}$$

where, $\mathbf{K}_1$, $\mathbf{K}_2$ and $\mathbf{K}_3$ are 1×4 control gain vectors. The equations of motion of the spacecraft with the reaction wheels have been formulated to accommodate such a P.I.D. control law.

The objects in the class **Body** export the transformation matrix, as well as the inertia matrix and the wrench vector to the *system* object.[3] They are used to update the elements in the *Inertia*, *Wrench*, and *Link* objects.

Joint

The bodies of a multibody system are connected to each other via joints. The two fundamental joints are the prismatic and revolute joints, which are developed from the generic joint element. The joint objects have a maximum of six degrees of freedom. For each joint, it is possible to provide external actuation that could be in the form of joint forces or torques. The joint objects export the transformation and Jacobian matrices, which are used for the formulation of the Natural Orthogonal Complement matrix in the object *System*.

The translatory motion of a *Prismatic Joint* can be expressed in one of three different coordinate systems depending on the nature of the motion viz. rectangular, cylindrical, and spherical. The two sets of Euler angles viz. $1-2-3$ Euler angles and $3-1-3$ Euler angles are available to describe the rotational motion of a *Revolute Joint*.[3,13]

Assembly

The process of assembling and solving the dynamic equations of motion of the multibody system are done in the classes **System** and **Solver**, respectively. The class **System** contains an object *System* that assembles the individual wrenches and inertia matrices of each body and combines them to form the equations of motion of the system. The equations of motion are then multiplied by the Natural Orthogonal Complement matrix in the *System* object. The Natural Orthogonal Complement matrix is built in the object *System*, by combining the spatial transformation matrices from the *Inward Link* and *Outward Link* objects and the spatial transformation and the local Jacobian matrices from the *Joint* object, for each of the bodies. Having gathered and computed the extended mass matrices and wrenches from the bodies, as well as the said Natural Orthogonal Complement matrix, the complete dynamic equations of motion of the system are formulated. The class **Solver** contains an object *Solver* that numerically integrates non-linear differential equations of motion of the system. The integrator is of Adams-Moulton type.[3]

Each object is implemented as a class module with a graphic icon in ROSE (Real-time Object-oriented Simulation Environment). The implementation in ROSE provides several benefits such as, automated code generation, interactive execution control, and rapid model development.[3] The actual modeling is achieved by the interconnection of the pre-casted object modules in an uncomplicated manner. Figure 6 depicts the interconnection of various objects in order to model a spacecraft mounted with three reaction wheels.

Simulations and Results

This section is concerned with the verification and simulation conducted using the designed objects. In

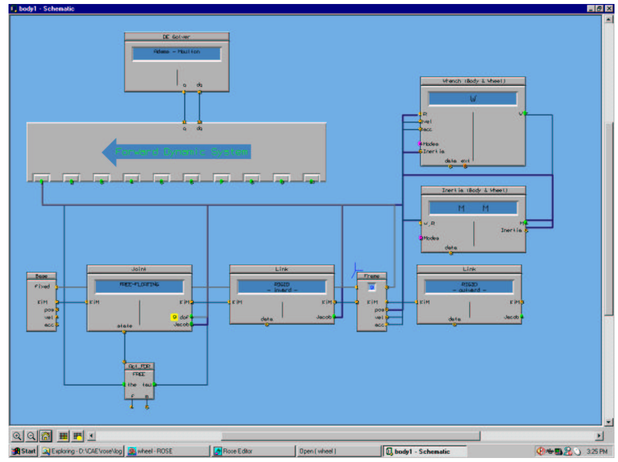

Fig. 6 ROSE model for a spacecraft with three reaction wheels

order to verify the accuracy of the designed objects, simulations were done using them and compared with results obtained using standard procedure, which were coded in MATLAB. So as to highlight the features and the control aspect of the designed objects, simulations for various examples have also been done.

The first set of simulations were done to verify the validity of the designed objects. The methodology used for coding the objects was that, initially objects for rigid bodies were coded and then the procedures were written for the dynamic coupling terms between the motherbody and the reaction wheel. The examples considered were the R R manipulator, body with two arms undergoing planar motion, and a spinning satellite In all the three cases , the responses got from MATLAB and ROSE are consistent.[12]

The Cassini spacecraft was launched on October 15, 1997 and after an interplanetary cruise of more than seven years, it will arrive at Saturn in February, 2005. Slew maneuvers were done on the Cassini spacecraft, using reaction wheels, on the seventy-fifth day of the year 2000 i.e. March 15, 2000. In order to corroborate the accuracy of the designed objects, the reaction wheels were given time histories, similar to the variation in spin rates of the reaction wheels located in the Cassini spacecraft. Telemetry data were available for the entire duration of the slews, at a frequency of once every four seconds.[16] The slew velocities of the Cassini spacecraft obtained via telemetry are plotted in figure 7. The aim of this validation test was to replicate the first slew maneuver about the Y-axis. The time histories that were given as inputs to the reaction wheels for a period of 1400 seconds are shown in figure 8, which are similar to the time histories of the reaction wheels during the actual slew maneuver of the Cassini spacecraft. As shown in figure 9, the spacecraft undergoes an approximate slew of -2.7×10^{-3} $rad./sec.$ about the Y-axis, which is the desired result. The angular velocity of the spacecraft about the Y-axis (ω_y) follows the correct trend of rising to an approximately constant slew rate about the Y-axis and then going back to zero at the end of the slew maneuver. During the slew maneuver ω_x and ω_z are zero. There are small differences between the actual slew velocity of the Cassini spacecraft and the generated simulation result due to the approximations made while prescribing the time histories to the reaction wheels.

Further simulations involving the reorientation and sequential slew maneuver were conducted to highlight the versatility of the designed objects, wherein the angular velocity of the motherbody could be specified and the reaction wheels would maintain that angular velocity/slew velocity of the motherbody.[12] In these simulations, it was shown that by varying the spin rate of the reaction wheels it was possible to reorient and also slew the spacecraft about specific axes.

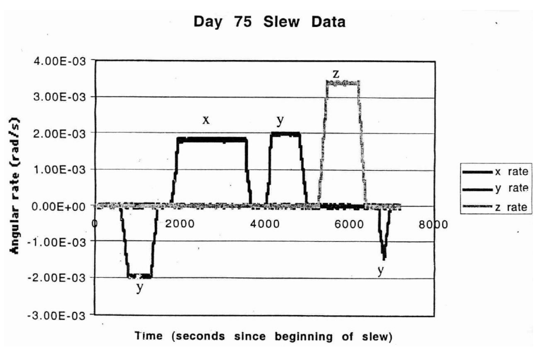

Fig. 7 Slew velocity of the Cassini spacecraft via telemetry data

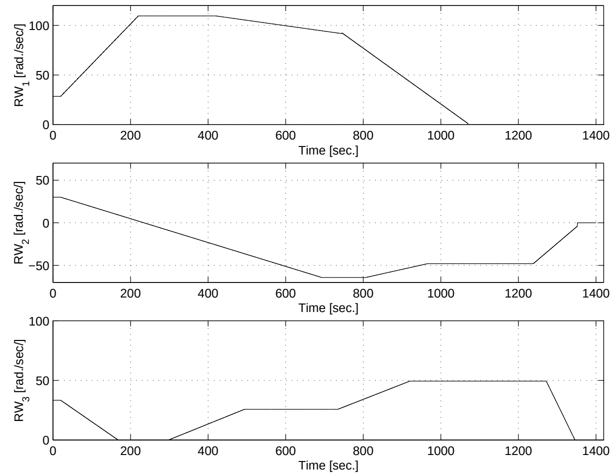

Fig. 8 Time histories of the reaction wheels during the slew maneuver

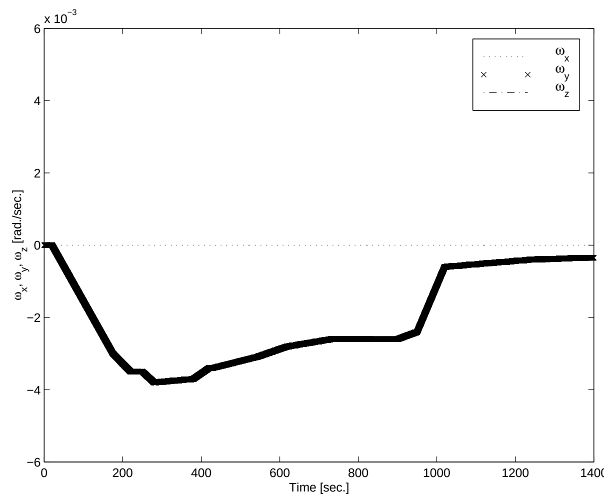

Fig. 9 Slew velocity of the Cassini spacecraft

Since attitude control of the spacecraft or satellite is of prime concern, reaction wheels are used as attitude control devices. Reaction wheels are referred to as momentum transfer devices. They compensate for the change in angular momentum of the system, when external torques are applied to the system. As a result, it is possible to maintain a constant angular momentum. The reaction wheels compensate for the change in angular momentum by varying its spin rate. In this section, the spin rate of the reaction wheels have been considered to be functions of the body quaternion of the motherbody. The objects have been designed such that spin rate of the reaction wheel can be in the form of a P.I.D. control law.

Clementine, the Deep Space Program Science Experiment spacecraft, was launched in January 1994 to map the surface of the moon.[17] The attitude control system requirements and constraints led to the need for small lightweight reaction wheels for the three-axis precision control system. The total mass of the three reaction wheels was 8.4 kg. The wheels were mounted such that their axes were mutually orthogonal. In the simulation presented here, the spacecraft was subjected to small initial perturbation of 0.02 $rad./sec.$, about all the three axes. The P.I. control law supplied to the wheels is given in equation (38). Each element of the 3×1 vector $\boldsymbol{\omega}_{wheel}$ represents the spin rate of one reaction wheel. It was possible to reduce the overshoot and settling time by applying this PI control law to the reaction wheels.[12]

$$\boldsymbol{\omega}_{wheel} = -65\hat{\mathbf{q}} - 55 \int \hat{\mathbf{q}}\, dt \qquad (38)$$

Attitude control of a spacecraft carrying two manipulators was also considered as an example.[12] The joint torque was applied to these appendages. A P.I.D. control law to the reaction wheel in order to maitain the attitude of the spacecraft.

Conclusion

In this paper, three reaction wheels, each spinning about a fixed axis and located in the spacecraft, are considered for the attitude control and stabilization of the motherbody. The development of objects that can enable simulation of a spacecraft containing reaction wheels has not been done prior to this work. Hence, the essence of this research was to develop a dynamics formalism which embraced the object-oriented concepts and addressed the dynamic simulation of a spacecraft with reaction wheels. This involved the modeling, designing, and coding of objects that would simulate the dynamic response of a complex multibody space system, with the motherbody containing reaction wheels.

The first step in this research work was to devise a mathematical model for the system under consideration. In order to frame the mathematical model of the system, the underlying kinematic relationships were first formulated. A variation of the Lagrangian dynamics and the principle of Natural Orthogonal Complement, in order to eliminate the kinematic constraints, were used to derive the dynamic equations of motion of the spacecraft coupled with the reaction wheels. Such a formulation technique has been proven to be computationally more efficient for complex multibody systems. The multibody system could be in the form of a spacecraft with multiple appendages, in an open chain configuration. The objects that simulate the dynamic response of the motherbody containing reaction wheels have been designed, so as to be part of a standard multibody system software package used in ROSE (Real-time Object-oriented Software Environment).

In order to ascertain the functionality and precision of the proposed objects, several validation tests were conducted. These representative examples of rigid body systems verified the definitiveness of the designedobjects by comparing the simulated responses with the results derived using standard procedure. Further validation of the designed objects was done by simulating a slew maneuver on the Cassini spacecraft, in order to verify the accuracy of the coded objects. The results obtained were similar to those available from the actual spacecraft. In order to highlight the adaptability and functionality of the coded objects, the next set of simulations conducted considered the motherbody containing a reaction wheel. It was shown for a representative multibody space system, that the motherbody's attitude is better controlled by a P.I.D. control law, applied to the reaction wheel, as opposed to reaction wheels spinning at a constant high spin rate.

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