2015 IEEE International Conference on Robotics and Automation (ICRA)
Washington State Convention Center
Seattle, Washington, May 26-30, 2015

Paramagnetic Microparticles Sliding on a Surface: Characterization and Closed-Loop Motion Control

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***Abstract*— In targeted therapy, clusters of drug carriers (nanoparticles and microparticles) could be in contact with a surface such as the lumen of blood vessels and the interior of the gastrointestinal tract. We study the motion characteristics of clusters of microparticles when they slide on a surface under the influence of weak oscillating magnetic fields (less than 11 mT). The oscillating magnetic fields exert a magnetic torque on the microparticles and allow them to oscillate, and hence overcome the static friction and slide on a surface. We characterize the frequency response of clusters of microparticles by applying oscillating magnetic fields with a frequency range of 0 Hz to 55 Hz, in the presence of a constant magnetic field gradient (0.9 T/m). Clusters of 3 to 4 and 5 to 9 microparticles achieve maximum sliding speeds of 1100 μm/s and 1150 μm/s, at oscillating magnetic fields of 30 Hz. In addition, we experimentally demonstrate closed-loop motion control of the clusters with maximum position error of 20 μm. Furthermore, we show that the magnetic field gradient required to drive a cluster of microparticles (with 3 to 4 microparticles) decreases by 75% in the presence of oscillating magnetic fields from 5 Hz to 50 Hz.**

I. INTRODUCTION

Clusters of superparamagnetic and ferromagnetic particles have the potential to be used as magnetic carriers in targeted therapy using an external source of magnetic field [1]-[4]. These particles would either move in a fluid or slide on a surface (Fig. 1), and then localize within the vicinity of a reference position under the influence of the controlled magnetic fields [5]. The magnetic field gradient could not be able to overcome the adhesive forces and the friction forces between the cluster and the surface. It becomes more difficult to pull and control using the magnetic field gradient as particles get smaller due to their magnetic dipole moment that decreases with their size.

Pawashe *et al.* have used time-varying magnetic fields provided by external electromagnets to achieve stick/slip motion of a microrobot [6]. This motion allowed the microrobot to move across various surfaces and surfaces with features. Time-varying magnetic fields have also been used to drive and steer sperm-shaped magnetic microrobots that swim [7], [8] and slide on a surface [9]. The motion of microrobots on surfaces has been also modeled by Nagy *et al.* using non-smooth multibody dynamics method with set-valued force laws [10]. However, microrobots or microparticles exhibit impact and interactions, and other complex phenomena such as sliding and stiction that are difficult to be modeled.

In this work, we characterize the sliding motion of clusters of paramagnetic microparticles inside water on the surface of a petri-dish. Motion of the cluster of microparticles is achieved by applying an oscillating weak magnetic field [11] to exert a magnetic torque on the magnetic dipole of the

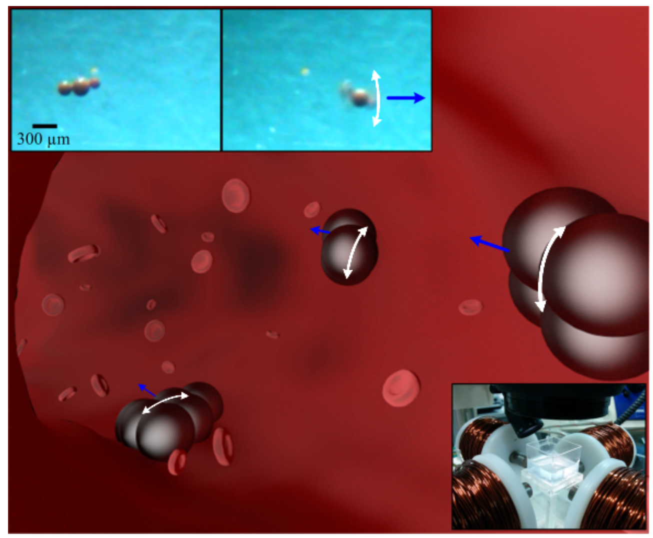

Fig. 1. Schematic representation of the sliding motion of clusters of paramagnetic particles inside a blood vessel. The particles slide on the lumen under the influence of the controlled oscillating magnetic fields (white arrows). The alternating magnetic field causes the cluster of particles to oscillate. These oscillations allow the microparticles to overcome the static friction between the particles and the lumen of the blood vessel. The gradient of the magnetic field pulls the particles and allows them to slide on the lumen towards a reference position. The blue arrows indicate the directions of the clusters. Experiments are done using clusters of microparticles (insets in the top-left corner) and magnetic fields are provided using an orthogonal array of electromagnetic coils (inset in the bottom-right corner). This schematic representation is designed using Blender (Blender 2.71, Blender Foundation, Entrepotdok, Amsterdam, The Netherlands).

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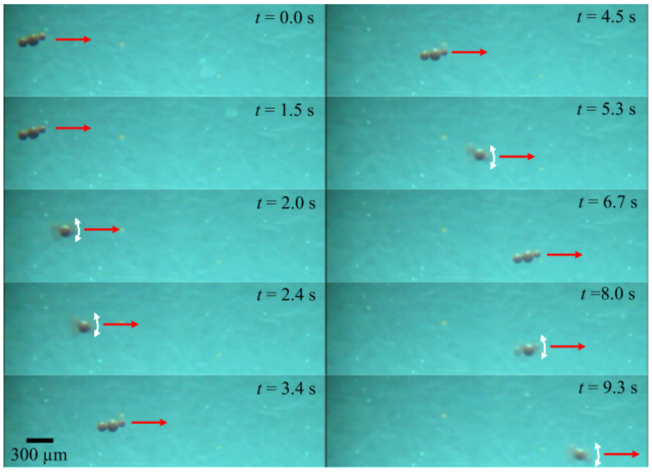

Fig. 2. Motion of a cluster of 3 paramagnetic microparticles under the influence of a constant pulling magnetic force in the absence and presence of the oscillating weak magnetic fields. The magnetic fields are oscillated after time instants, $t = 1.5$ seconds, $t = 4.5$ seconds, and $t = 6.7$ seconds, respectively. The oscillation allows the cluster to overcome the static friction with the petri dish and move using the constant magnetic field gradient (0.9 T/m). The cluster moves at an average speed of 300 μm/s when the oscillating magnetic field is applied at 30 Hz. The red and white arrows indicate the constant magnetic force and the oscillating magnetic fields, respectively. *Please refer to the accompanying video that demonstrates the motion of the cluster of microparticles.*

Fig. 3. Motion of a cluster of 9 paramagnetic microparticles under the influence of a constant pulling magnetic force in the absence and presence of the oscillating weak magnetic fields. The magnetic fields are oscillated after time instants, $t = 1.5$ seconds, $t = 4.5$ seconds, and $t = 6.7$ seconds, respectively. The oscillation allows the cluster to overcome the static friction with the petri dish and move using the constant magnetic field gradient. The cluster moves at an average speed of 300 μm/s when the oscillating magnetic field is applied at 30 Hz. The red and white arrows indicate the constant magnetic force and the oscillating magnetic fields, respectively. *Please refer to the accompanying video that demonstrates the motion of the cluster of microparticles.*

cluster. The oscillations that are caused by the field allow the cluster to overcome the static friction and decreases the required magnetic field gradient required to pull the cluster towards a reference position. The characterization is done using an orthogonal array of electromagnetic coils that allows us to apply oscillating magnetic fields (with a frequency range of 0 Hz to 55 Hz) and a constant magnetic field gradient (0.9 T/m) within the center of the workspace of the electromagnetic configuration.

The remainder of this paper is organized as follows: Section II provides the modeling and frequency response characteristics of clusters with 3 to 4 microparticles and 5 to 9 microparticles. The magnetic-based motion control experimental results of the cluster of microparticles are included in Section III. Section IV provides a discussion pertaining to the self-assembly of sperm-shaped microrobots using microparticles and the flagellated swim under the influence of oscillating magnetic fields. Finally, Section V concludes and provides directions for future work.

II. Characterization of the Sliding Microparticles

An almost uniform weak magnetic field could not generate enough pulling magnetic force on the cluster of microparticle to overcome the drag and friction forces. Therefore, we apply an oscillating magnetic field as a low magnitude of torque will ultimately align the cluster along the field lines, and hence overcome the static friction force between the cluster and the surface. This static friction force can not be overcome by the magnetic field gradient. A proof-of-concept experiment is provided in Fig. 2. This representative experiment shows that the pulling magnetic force that is exerted on the magnetic dipole of the cluster cannot overcome the friction force. This force is overcome once the magnetic fields are oscillated after time instant, $t = 1.5$ seconds. The oscillating magnetic fields are not applied after the time, $t = 2.4$ seconds, and we observe that the magnetic field gradient cannot pull the cluster. We repeat this procedure by applying the oscillating magnetic fields again after time, $t = 4.5$ seconds, and the cluster breaks free of the static friction and slide on the surface of the petri-dish at an average speed of 300 μm/s. Motion of the cluster of microparticles on a surface with static coefficient of friction (μ_{s}) is given by

$$\nabla(\mathbf{m}(\mathbf{P}) \cdot \mathbf{B}(\mathbf{P})) + \mathcal{F}_{\mathrm{c}}\eta\dot{\mathbf{P}} + \mu_{\mathrm{s}} m_{\mathrm{c}} \mathbf{g} = 0, \quad (1)$$

where $\mathbf{m}(\mathbf{P})$ is the magnetic dipole moment of the cluster of microparticles and $\mathbf{B}(\mathbf{P}))$ is the induced magnetic field at point $(\mathbf{P})$. Further, $\mathcal{F}_{\mathrm{c}}$, and η are the shape factor of the cluster and the fluid dynamic viscosity (1 mPa.s), respectively. Furthermore, m_{c} is the mass of the cluster and $\mathbf{g}$ is the acceleration due to gravity. The cluster of microparticles break free and slides on the surface when the magnetic force overcomes the drag and static friction forces. This requires relatively large magnetic field gradients. Instead, we apply weak (less than 11 mT) oscillating magnetic fields at different frequencies to exert a magnetic torque and align the cluster along the oscillating field lines. This oscillation overcomes the static friction and allows the cluster to move since the kinetic friction is less than the coefficient of static

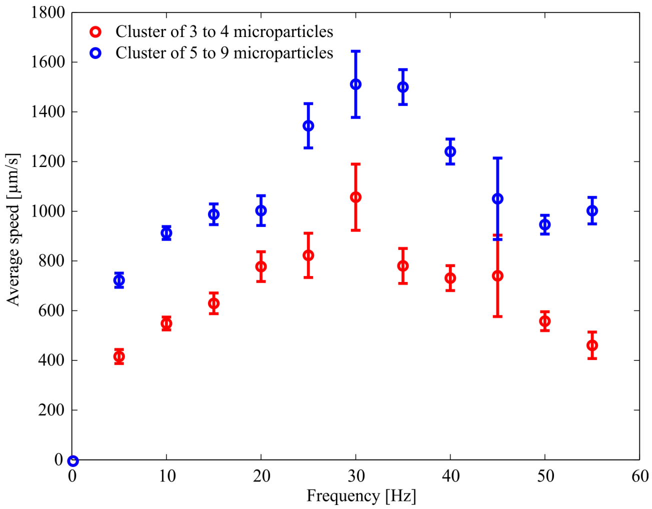

Fig. 4. Frequency response of clusters of paramagnetic microparticles. Clusters are formed using 3 to 4 microparticles and 5 to 9 microparticles. Clusters with more than 9 microparticles break into smaller clusters. Therefore, their frequency response is not calculated. The average speed is calculated using 5 trials at each frequency. The speed is calculated when the clusters move along x-axis. The clusters have zero speed under the influence of magnetic field gradient without oscillation. Motion of the clusters is observed when the oscillating magnetic fields are applied in the presence of a constant magnetic field gradient of 0.9 T/m.

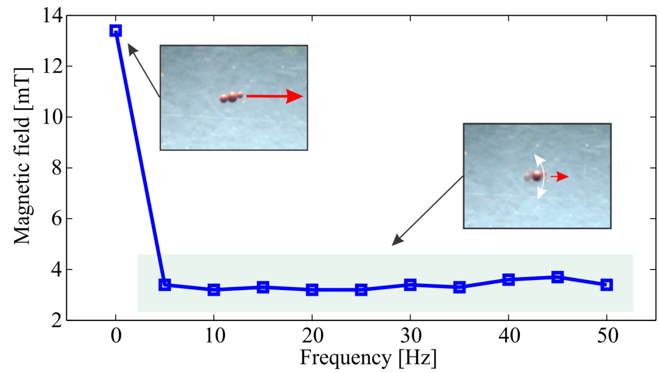

Fig. 5. The minimum magnetic field required to pull the cluster versus the oscillating field frequency. The field is gradually increased to increase the pulling magnetic force. A magnetic field magnitude of 13.4 mT is calculated when no oscillations are applied. The experiment is repeated from 5 Hz to 50 Hz and the average magnitude of the magnetic field measured is 3.37 mT. This shows a 75% decrease in the magnetic field required to slide the clusters when the oscillations are applied. Magnetic fields are measured using a calibrated 3-axis digital Teslameter (Senis AG, 3MH3A-0.1%-200mT, Neuhofstrasse, Switzerland

friction for the same materials [12]. Therefore, (1) can be rewritten as

$$\nabla(\mathbf{m}(\mathbf{P}) \cdot \mathbf{B}(\mathbf{P})) + F_{\mathrm{c}}\eta\dot{\mathbf{P}} + \mu_{\mathrm{k}} m_{\mathrm{c}}\mathbf{g} = 0, \quad (2)$$

where μ_{k} is the coefficient of kinetic friction. Since $\mu_{\mathrm{k}} < \mu_{\mathrm{s}}$, the driving magnetic force required to pull the cluster in (2) is less than that in (1). We use a proportional control system to control the microparticles. Therefore (2) is given by

$$\mathbf{Ke} + F_{\mathrm{c}}\eta\dot{\mathbf{e}} + \mu_{\mathrm{k}} m_{\mathrm{c}}\mathbf{g} = 0, \quad (3)$$

where $\mathbf{e}$ is the calculated position tracking error, and $\mathbf{K}$ is the proportional gain matrix. We characterize the effect of the oscillating field frequency on the motion of the cluster of microparticles and the number of microparticles within the cluster using an electromagnetic system.

A. Electromagnetic System

Our characterization and motion control experiments are done using an electromagnetic system with closed-configuration, as shown in Fig. 3. Four electromagnetic coils are independently supplied with current inputs using electric drivers (MD10C, Cytron Technologies Sdn. Bhd, Kuala Lumpur, Malaysia) and controlled via an Arduino control board (Arduino UNO - R3, Arduino, Memphis, Tennessee, U.S.A) and Simulink (MathWorks, Natick, Massachusetts, U.S.A). Each coil has inner-diameter, outer-diameter, length, and number of turns of 10 mm, 50 mm, 60 mm, and 1200 turns, respectively. Each electromagnetic coil provides maximum magnetic field and magnetic field gradient of 14.5 mT and 0.9 T/m, respectively, at the center of the workspace (10 mm×10 mm) of the system. Position of the cluster of microparticles is determined using a stereo microscopic system (Stemi 2000-C, Carl Zeiss Microscopy, LLC, New York, U.S.A). The electromagnetic coils are used to provide oscillating magnetic fields to characterize the frequency response of the cluster of microparticles.

B. Frequency Response of Clusters of Microparticles

Although any oscillation of the cluster will ultimately result in a transition between static to kinetic friction, and hence allow us to pull the cluster using relatively less magnetic field gradient, we study the effect of the oscillating magnetic fields on the speed of the cluster. This characterization is done under the influence of a constant magnetic field gradient of 0.9 T/m throughout a frequency range of 0 Hz to 55 Hz. This frequency range is devised based on the speed of the clusters of microparticles. The maximum speed of the clusters is decreased by approximately 50% at a frequency of 50 Hz. The frequency response characterization is done using clusters with 3 to 4 microparticles and clusters with 5 to 9 microparticles. Clusters with more than 9 microparticles break into smaller clusters and therefore their response is not compared to the mentioned clusters. The frequency response is provided in Fig. 4. The average speed of the cluster increases as we increase the frequency of the oscillating magnetic fields. A decrease in the average speed of the clusters happens after 30 Hz for both clusters. The cluster of 5 to 9 microparticles possesses greater net magnetic dipole moment than the cluster of 3 to 4 microparticles. Therefore, its maximum speed is 30% greater than the maximum speed of the cluster of 3 to 4 microparticles. The frequency response is determined by moving the clusters of microparticles along x-axis, as shown in Fig. 6. In this representative frequency response experiment, the cluster of microparticles move at an average speed of 890 μm/s, under

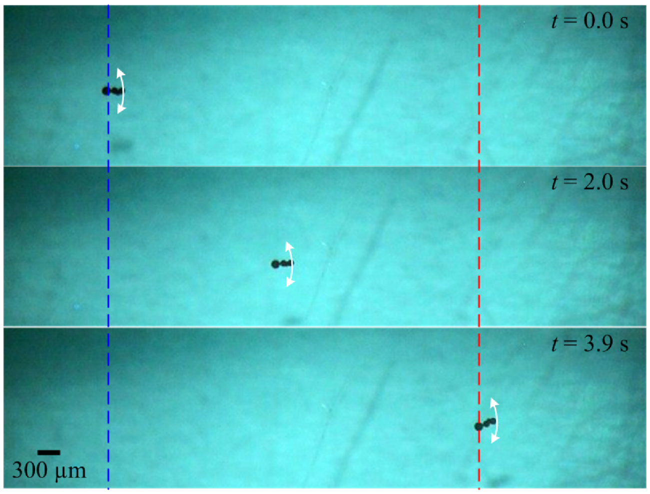

Fig. 6. Motion of a cluster of microparticles under the influence of oscillating magnetic field at 40 Hz. The cluster slides and moves at an average speed of approximately 3 body lengths per second. The white arrow represents the oscillating magnetic fields, whereas the dashed blue and red lines indicate the starting and ending positions of the cluster, respectively. In this experiment microparticles are contained in water and slide on the surface of a petri dish. *Please refer to the accompanying video that demonstrates the motion of the cluster of microparticles.*

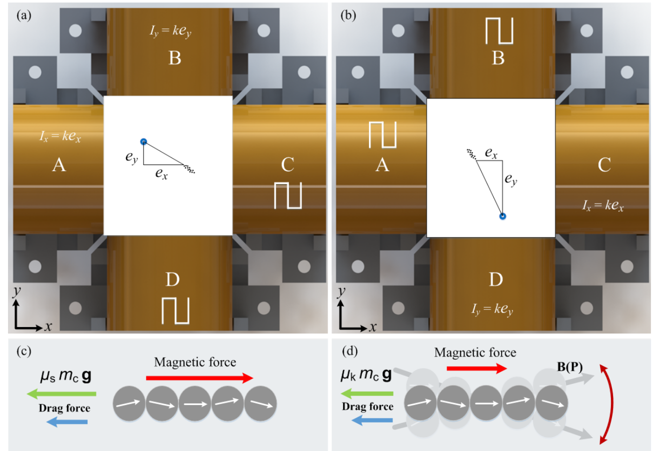

Fig. 7. Schematic representation of the control strategy used to allow the cluster to swim or slide on a surface towards the reference position (small blue circle). Further, e_x and e_y represent the position tracking errors along x- and y-axis, respectively. The letters A, B, C, and D indicate the electromagnetic coils. The currents at the electromagnets producing the uniform fields are I_x and I_y . (a) The cluster is oriented towards the top left quadrant by electromagnets A and B. Electromagnets C and D provide oscillating fields to oscillate the cluster. (b) Electromagnets C and D provide uniform magnetic fields in the direction of the reference point. Electromagnets A and B generate oscillating magnetic fields. (c) The forces exerted on a cluster without oscillating magnetic fields. (d) The forces exerted on the cluster under the influence of the oscillating fields.

the influence of oscillating magnetic fields at 40 Hz.

The increase in the average speed of the clusters with the increasing frequency is attributed to the rigid flagellated oscillations of the clusters. Majority of the clusters that are used throughout our experimental work have a longitudinal structure with one or two parallel rows of microparticles. The oscillating magnetic fields not only allow the cluster to overcome the static friction but also slide faster on the surface of the petri-dish, as shown in Fig. 4. After 30 Hz, the cluster of microparticles can no longer align along the magnetic field lines. Therefore, the oscillation amplitude decreases, and hence the propulsion force due to the rigid flagellated oscillations decreases.

We also investigate the effect of the oscillating magnetic fields on the magnitude of magnetic field gradient required to move the cluster of microparticles. In this experiment, an increasing magnetic field gradient is applied to pull clusters of microparticles along the x-axis. The magnetic field gradient is increased gradually by increasing the applied current input till a sliding motion is observed. Fig. 5. shows that the magnetic field required to move the cluster without applying oscillations is calculated to be 13.4 mT. We repeat this experiment 10 times with a frequency range of 5 Hz to 50 Hz and the average magnetic field calculated is 3.37 mT. Therefore, sliding motion of the microparticles can be achieved using 75% less magnetic fields (when these field are oscillated) than magnetic fields without oscillation.

III. Motion Control of the Sliding Microparticles

Motion control of the sliding microparticles is achieved by orienting the oscillating magnetic field lines towards a reference position within the workspace of our electromagnetic system. The cluster of microparticles are controlled using using four electromagnets along the x- and y-axis. The orientation of the cluster is controlled using two electromagnets while the oscillating field is produced using the other two electromagnets. We calculate the position tracking errors between the cluster and the reference position using:

$$e_x = x_{\text{ref}} - x \quad \text{and} \quad e_y = y_{\text{ref}} - y, \tag{4}$$

where e_x and e_y are the position tracking error along x- and y-axis, respectively. Also, x and y are the position of the cluster along x- and y-axis, respectively. Further, x_{ref} and y_{ref} are the components of the fixed reference position. Two electromagnets produce a uniform field to pull the microparticles towards the reference position. The other two electromagnets produce the oscillating fields and are always in the opposite direction to the direction of movement of the cluster. The position of the cluster relative to the reference determines the electromagnets which produce the uniform field. The magnitude of the current supplied to the electromagnets producing the uniform field is given by:

$$I_x = ke_x \quad \text{and} \quad I_y = ke_y, \tag{5}$$

where I_x and I_y are the current magnitudes of each electromagnet and k is the proportional gain. Fig. 7. shows a schematic representation of the control strategy used to allow the cluster to slide towards the reference position. In Fig. 7(a) electromagnets D and C provide oscillating fields while electromagnets A and B produce uniform magnetic

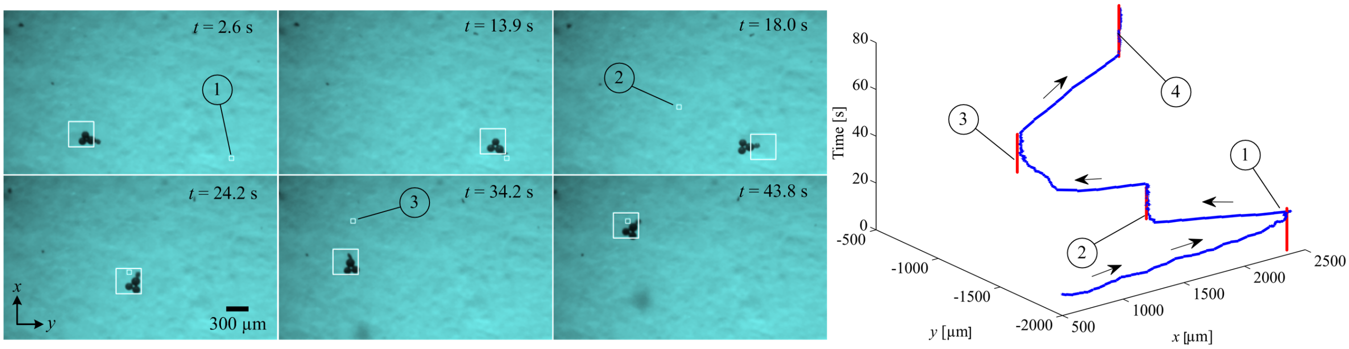

Fig. 8. Point-to-point closed-loop motion control of a cluster of microparticles towards 3 reference positions. The magnetic fields are oscillating at frequency of 10 Hz. The average speed of the cluster of microparticles is calculated to be 223 μm/s, whereas the maximum position error is 20 μm in the steady-state. The vertical red lines indicate the 3 reference positions (represented by the small white squares in the representative snap-shots). The blue lines indicate the trajectory of the cluster of microparticles. The white square indicates the position of the microparticles and is assigned using our feature tracking algorithm. In this experiment microparticles are contained in water and slide on the surface of a petri dish. *Please refer to the accompanying video that demonstrates the motion of the cluster of microparticles.*

fields with magnitudes of I_x and I_y, respectively, to move the cluster towards the reference position. Figs. 7(c) and (d) show schematic representations of the forces exerted on the cluster in the absence and presence of oscillating magnetic fields.

A representative point-to-point motion control result is provided in Fig. 8. Once a reference position is given to the closed-loop control system, the oscillating magnetic field orient towards it. This allows the cluster to break free and slide towards 4 different reference positions. In this experiment, oscillating magnetic fields of 10 Hz is applied and the cluster moves towards the reference potion at an average speed of 223 μm/s. The maximum position tracking error in this experiment is calculated to be 20 μm. *Please refer to the accompanying video that demonstrate our point-to-point closed-loop motion control of a cluster of microparticles using oscillating magnetic fields.*

We also achieve motion control of a circular trajectory (with diameter of 1904 μm) by moving the cluster of microparticles towards 20 way-points along the trajectory, as shown in Fig. 9. In this experiment, the oscillating magnetic fields are applied at frequency of 5 Hz and the average speed of the cluster of microparticles is calculated to be 60 μm/s. The difference between the average speed of the cluster of microparticles in the frequency response characterization and the closed-loop control is due to the direction of the path of the cluster. The frequency response is determined when the clusters are pulled along the x-axis, whereas the point-to-point closed-loop control system moves the clusters in different directions using different magnetic field gradients. *Please refer to the accompanying video that demonstrates the closed-loop control of the cluster of microparticles along a circular trajectory.*

IV. Discussion

The coulomb forces between microparticles allow for the formation of clusters. The configuration of the microparticles within these clusters affects the net magnetization of the cluster, and hence decrease or increase the magnetic force exerted on its dipole. We observe that clusters with longitudinal configurations and single row of microparticles move at higher average speeds than clusters with other configurations. We attribute this behaviour to two reasons. First, clusters of single row of microparticles have higher net magnetization, as opposed to clusters with random configuration of microparticles. The higher net magnetization results in a greater magnetic force for the same external magnetic field gradient. Second, clusters of single row of microparticles exhibit flagellated motion when oscillating magnetic fields are applied. The frequency response of the cluster of microparticles indicates a decrease in the speed above frequency of 30 Hz. In addition, the frequency response shows that the speed of the cluster depends on the frequency of the oscillating fields under the influence of a constant magnetic field gradient. Therefore, the speed of the cluster is affected by the amplitude of its flagellated motion. Sperm-shaped magnetic microrobots (which consist of magnetic head and a continuous flexible tail) [7], [16] show similar response to weak oscillating magnetic fields. The difference between these microrobots and cluster of single row of microparticles is the lumped nature of the cluster. Therefore, clusters of microparticles can be used to assemble microrobots that depend on flagellar propulsion using weak oscillating magnetic fields.

V. Conclusions and Future Work

We experimentally demonstrate that using weak oscillating magnetic fields allows clusters of paramagnetic microparticles to break free of the static friction force and slide on a surface. The oscillating magnetic fields wiggle to make a transition from static to dynamic friction. Therefore, lower magnetic field gradients are required to pull the clusters of microparticles, as opposed to pulling with magnetic fields without oscillation. The magnetic field gradient required to pull a cluster of 3 to 4 microparticles is 75% less than that required to pull the same cluster when the fields are oscillated

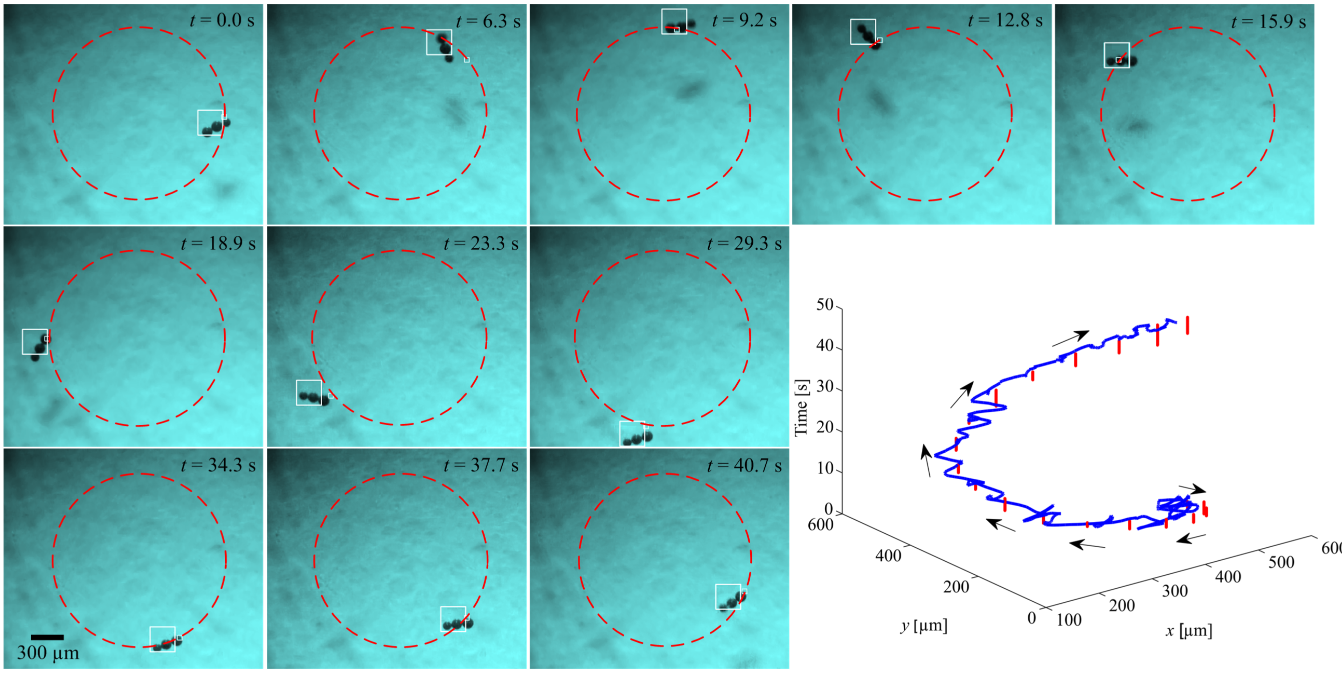

Fig. 9. Closed-loop motion control of a cluster of microparticles along a circular trajectory with diameter of 1904 μm. 20 way-points are defined along the circular trajectory and are shown using the vertical red lines. The way points are also indicated using the small white squares. The cluster is oscillated at a frequency of 5 Hz in the presence of a constant magnetic field gradient. The average speed of the cluster along this trajectory is calculated to be 60 μm/s. The blue line represent the trajectory of the microparticles and the dashed-red circle indicates the refernece circular trajectory. The black arrows and the white square indicate the direction of the cluster and the position of the cluster. The white square is assigned using our feature tracking software. In this experiment microparticles are contained in water and slide on the surface of a petri dish. *Please refer to the accompanying video that demonstrates the motion of the cluster of microparticles along the circular trajectory.*

at frequencies from 5 Hz to 50 Hz. We characterize the frequency response of clusters of 3 to 4 microparticles and 5 to 9 microparticles, and show that they can only be pulled when the magnetic fields are oscillated. We also show that they move at maximum speeds of 1050 μm/s and 1550 μm/s at frequency of 30 Hz for clusters of 3 to 4 and 5 to 9 microparticles, respectively.

As part of future work, a sperm-shaped microrobot will be formed using single row of microparticles and weak oscillating magnetic fields will be used for propulsion and steering. We will also investigate the self-assembly of sperm-shaped microrobots using microparticles and nanoparticles.

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