Abstract — The presence of force feedback in medical instruments has been proven to reduce tissue damage. In order to provide force feedback, information about the interaction forces between the instrument and the environment must be known. Direct measurement of these forces by commercial sensors is not feasible due to space limitations. Thus, in this study we propose to estimate the interaction forces using strain measurements from Fiber Bragg Grating (FBG) sensors. These measurements can also be used for shape sensing and as a result both force and shape can be sensed simultaneously. For force sensing two models are proposed and compared. The first is based on a Rigid Link approximation, while the second uses the Cosserat rod theory. The models are validated experimentally using a tendon-driven continuum manipulator that is subjected to forces at the tip. The force estimates from the models are compared to the measurements from a commercial force sensor. Mean absolute errors of 11.2 mN (6.9%) and 15.9 mN (8.5%) are observed for the Rigid Link model and Cosserat model, respectively.

I. INTRODUCTION

Many of the instruments currently used in medical procedures have a mechanical design similar to continuum manipulators [1]. Examples of such instruments are colonoscopes, endoscopes and other flexible catheters for procedures such as cardiac surgery and bronchoscopy. Some continuum manipulators have been developed specifically for medical applications. These include multi-backbone system for throat surgery, concentric tube active cannula for cardiac surgery and steerable probe for neurosurgery [2]–[4]. These manipulators can be easily miniaturized and they provide a larger workspace compared to rigid tools, thus they are ideal for minimally invasive procedures [1]. However, the disadvantage of using such manipulators is the loss of force information at the tip. Having accurate knowledge about the interaction forces between the manipulator and tissue is important for the outcome of the procedure [5]. It can be used to provide surgeons with force feedback, thereby enabling more precise manipulation of the tissue. Sensing forces accurately at the manipulator tip is challenging because the available space does not allow integration of commercially-available force sensors. Considering further miniaturization of manipulator size in the future, there is a need for alternative methods for sensing interaction forces on manipulators for medical applications.

A number of studies have proposed methods to identify the interaction forces on manipulators without measuring them directly. For example, Xu and Simaan presented a method that used the deflected shape of the manipulator shaft to estimate the force at the tip [6], [7]. Rucker and Webster estimated the tip force using pose measurements and a kinematic static model of the continuum manipulator with measurement uncertainty [8]. They described the tip force as a state of the system, and an extended Kalman filter was used to estimate the system states from the noisy end-effector pose measurements. Back et al. estimated the tip force using shape of the catheter and the Cosserat rod model [9]. Lastly, Khosnam et al. used the curvature of a catheter, determined from camera images, in combination with a kinematic model to estimate the tip contact force [10].

Another approach to force estimation is to sense the strains on the manipulator directly using sensors. The benefit of using an independent sensor for force is that the robustness of the system to sensor failure will improve due to redundancy and in theory direct measurement of strain will lead to more accurate force estimation. A suitable sensor for medical applications is the Fiber Bragg Grating sensor because it is small in size, sterilizable, biocompatible, highly sensitive to strain, and compatible with medical imaging modalities [11]. The FBG sensors can be embedded in instruments.
manipulators because of mechanical constraints, an example

They have also been helically wrapped around the shaft

do invasive neurosurgery in order to measure interaction forces

for angioplasty, gastric endoluminal surgery or minimally

needle shape during insertion into soft-tissue phantoms,

and for shape reconstruction of a tendon-driven continuum

manipulator in free-space [18], [19]. Strain measurements

from at least 3 co-located FBG sensors are required in order

to calculate the magnitude (∥κ∥) and direction (ϕ) of

the curvature vector at a specific location (sk, k ∈ Z+)

along the manipulator shaft (Figure 2). Each FBG sensor measures

a strain (ε), where * ∈ (a, b, c), that is given by

ε∗ (sk) = ∥κ(sk)∥ δ∗ (sk) + ε0,

= ∥κ(sk)∥ r∗ cos(ϕ(sk) − α∗) + ε0, (1)

where δ is the distance from the fiber center to the neutral

bending axis, r∗ is the distance of the fiber center to the

center of the manipulator, α∗ is the angle of the fiber with

respect to x-axis at the manipulator cross-section, κ(sk) ∈ R is

the curvature vector and ϕ(sk) ∈ R is the direction of

bending (Figure 2). Each sensor is assumed to have a

common offset (ε0) in the measured strain, that can be caused

by a change in the environmental temperature or an axial

force along the manipulator shaft. The three unknowns (i.e.,

∥κ∥, ϕ and ε0) are solved from the strains measured by

the three co-located FBG sensors. The curvature magnitude

(∥κ(sk)∥) and the bending direction (ϕ(sk)) can be used to

define the curvature vector (κ(sk)) at each sensor location:

κ(sk) = [κx(sk) κy(sk) κz(sk)] = ∥κ(sk)∥ [cos(ϕ(sk))] [sin(ϕ(sk))]. (2)

Interpolation of the curvature components (κx(sk), κy(sk), κz(sk))
between each of the FBG sensor locations is performed in

order to obtain the curvature vector (κ(s)) at every location

along the manipulator shaft (Figure 2). The bending direction

(ϕ(s)) along the shaft, is equal to the direction of the curva-
ture vector. The curvature and the bending direction defines

the orientation, which can be used to evaluate the tangent

vector (t(s)) of the curve and t(s) = dr/ds, where r(s) ∈ R3

is the position vector of the curve. Hence, manipulator shape

can be reconstructed by numerical integration of the tangent

vector. The curvature and direction of bending information

will be required by both the Rigid Link and Cosserat model.

III. RIGID LINK MODEL

In rigid link robots contact forces/torques at the end-
effector are estimated using joint forces/torques [20] [21].
The continuum manipulator is modeled as a rigid link robot

with revolute joints [22]. The interaction forces at the tip are
determined based on the joint torques in the model.

A. Kinematics

The continuous shape of a manipulator is approximated

by a serial chain of n rigid links, connected by n revolute

joints (Figure 3). This method can be used to describe non-

constant curvature bending, that occur in flexible instruments

subjected to external loading. Link orientation is described

by three consecutive rotations, thus results in the following

rotation matrix of the i-th link with respect to the i - 1 link:

Ri-1 = Rz(qz,i)Ry(qy,i)Rx(-qz,i), (3)

where Ri ∈ SO(3) and Rq ∈ SO(3) are rotation ma-

trices about the z-axis and y-axis of the rotated frame,
respectively (Figure 3). The two joint angles that determine the rotations in (3) are related to the direction of bending ($q_{\varphi,i} \in \mathbb{R}$) and the amount of bending ($q_{\theta,i} \in \mathbb{R}$). In order to describe the manipulator elasticity using the Rigid Link model, each joint is assigned a flexural stiffness that is related to the bending stiffness. In the next sub-section, manipulator statics is used to relate manipulator shape to the manipulator tip wrenches which is the result from actuation and from external loading.

**B. Statics**

In order to calculate manipulator configuration, the joint angles of the Rigid Link model need to be related to the loads that act on the manipulator. In static equilibrium, the loads that act on the manipulator are balanced by the torques generated in the joints. The joint torques are not generated by motors, but are the result of manipulator bending. The joint torque vector ($\tau_i$) at the $i$-th joint is given by the following relation:

$$
\tau_i = K_{\theta,i}q_{\theta,i},
$$

where, $K_{\theta,i}$ is the flexural stiffness, $q_{\theta,i}$ is the amount of bending (Section II), and $\tau_i \in \mathbb{R}$ is the magnitude of the bending moment at the location of the joint. The bending moment ($m(s_i) \in \mathbb{R}^3$) at the $i$-th joint is the sum of the contribution of an actuation moment and an external force (Figure 3):

$$
m(s_i) = m_{ac} + r_F(s_i) \times F_{ext},
$$

where, $m_{ac} \in \mathbb{R}^3$ is the actuation moment, $F_{ext} \in \mathbb{R}^3$ is the external force, and $r_F(s_i) \in \mathbb{R}^3$ is the vector from the external force contact point to joint $i$. For a continuum manipulator with $n$ rigid links connected by $n$ joints, the joint torque vector ($\tau \in \mathbb{R}^n$) can be written as

$$
\tau = \tau_{ac} + \tau_{ext},
$$

where $\tau_{ac} \in \mathbb{R}^n$ and $\tau_{ext} \in \mathbb{R}^n$ are the joint torques due to (internal) actuation and (unknown) external forces, respectively. We assume that actuation generates a pure moment at the end of the manipulator (Figure 3). The joint torques due to the actuation moment ($m_{ac} \in \mathbb{R}^3$) can be calculated using

$$
\tau_{ac} = J_m^T \begin{bmatrix} 0_3 \\ m_{ac} \end{bmatrix} = J_m^T m_{ac},
$$

where $J_m \in \mathbb{R}^{6 \times n}$ is the manipulator Jacobian, $0_3 = [0 \ 0 \ 0]^T$, and $m_{ac} \in \mathbb{R}^6$ denotes the actuation wrench. The contribution to the joint torque vector in (6) is given by

$$
\tau_{ext} = J_{cp}^T \begin{bmatrix} F_{ext} \\ 0_3 \end{bmatrix} = J_{cp}^T m_{ext},
$$

where $J_{cp} \in \mathbb{R}^{6 \times n}$ is the contact point Jacobian. The above formulation will be used in combination with shape information ($q_{\varphi,i}$ and $q_{\theta,i}$) derived from FBG sensor measurements. The next section presents the rigid link model fitting based on the shape information.

**C. Rigid Link Model Fitting**

The model consists of $n$ joints and $n$ rigid links, each joint has two degrees of freedom, one related to the bending and the other related to the direction of bending. The joint angles can be determined from the curvature vector ($\kappa(s) \in \mathbb{R}^2$) and bending direction ($\varphi(s) \in \mathbb{R}$), which are calculated from FBG sensors (Section II). The joint angles that define the configuration of the Rigid Link model can be related to the curvature vector. Integrating the curvature magnitude gives the slope ($\theta(s) \in \mathbb{R}$) along the shaft:

$$
\theta(s) = \int_0^s \|\kappa(s)\|ds.
$$

The joint angle related to manipulator bending can be calculated from manipulator slope as

$$
q_{\theta,i} = \frac{1}{2} \Delta \theta_{i-1} + \frac{1}{2} \Delta \theta_i,
$$

where $\Delta \theta_i$ is the change in manipulator slope between two consecutive joints:

$$
\Delta \theta_i = \theta(s_{i+1}) - \theta(s_i).
$$

The joint angle related to the bending direction equals the curvature direction at the location of the $i$th joint ($q_{\varphi,i} = \varphi(s_i)$). Manipulator configuration is now fully defined given the curvature vector from the FBG sensor measurements. This allows the derivation of the manipulator Jacobian, which is used in the next section to estimate the unknown external forces.

**D. Contact Force Estimation**

The joint torque due to the unknown external load is given by $\tau_{ext} = \tau - \tau_{ac}$, where $\tau$ is determined using (4) and $\tau_{ac}$ is from (7). Thus, combining (4) and (7)

$$
\tau_{ext} = \begin{bmatrix} \tau_1 \\ \vdots \\ \tau_n \end{bmatrix} = \begin{bmatrix} K_{\theta,1}q_{\theta,1} - \tau_{ac,1} \\ \vdots \\ K_{\theta,n}q_{\theta,n} - \tau_{ac,n} \end{bmatrix},
$$

The contact points are assumed to be known for all (unknown) external loads, such that the Jacobian ($J_{cp}$) for the contact point can be determined using the forward kinematics of the Rigid Link model. In the case of a single external
load \((\mathbf{F}_{ext})\) at the manipulator tip, the contribution to a joint torque vector from the load is given by
\[
\tau_{ext} = \mathbf{J}_{cp}^T \left[ \frac{\mathbf{F}_{ext}}{\mathbf{0}_3} \right],
\]
where \(\mathbf{J}_{cp} \in \mathbb{R}^{6 \times n}\) and \(\mathbf{F}_{ext} \in \mathbb{R}^3\). For multiple external loads \(\mathbf{J}_{cp} \in \mathbb{R}^{6m \times n}\), where \(m\) is the number of external loads. Since the manipulator Jacobian is often non-square, the external force is estimated using the pseudoinverse of the Jacobian
\[
\left[ \frac{\mathbf{F}_{ext}}{\mathbf{0}_3} \right] = (\mathbf{J}_{cp}^T)^+ \tau_{ext},
\]
where \((.)^+\) denotes the Moore-Penrose pseudoinverse, and \(\tau_{ext}\) is calculated from \((12)\). Thus, the external force \((\mathbf{F}_{ext})\) is estimated using the rigid Link model. The next section presents the Cosserat model that can also be utilized to estimate the external force.

IV. COSSERAT MODEL

The Cosserat rod theory presents an geometrically exact model for a flexible rod, this is the motivation for the Cosserat model. The model presented in the paper is applicable to manipulators that have small cross section area compared to their length and are not subjected to torsion or axial forces. The force at the tip of the continuum manipulator can be estimated based on the shape information calculated from the strain measurements of the FBG sensors. 

A. Kinematics

The manipulator kinematics is based on a continuous transformation that is a function of the arc length [23]. In general, the transformation is dependent on the strains and shear stress acting on the manipulator, however given the assumption that the manipulator is not subjected to torsion and axial forces, the kinematics can be simplified such that the position \((\mathbf{r}(s) \in \mathbb{R}^3)\) as a function of the arc length \((s \in \mathbb{R})\) can be determined by solving the following:
\[
\begin{align*}
\mathbf{r}'(s) &= \mathbf{R}(s) \mathbf{e}_3, \\
\mathbf{R}'(s) &= \mathbf{R}(s) \hat{\mathbf{u}}(s),
\end{align*}
\]
where \(\mathbf{r}'(s)\) is the derivative with respect to \(s\), \(\mathbf{R}(s) \in SO(3)\) is the rotation matrix and it represents the change in curvature with respect to the arc length, \(\mathbf{e}_3 = [0 \ 0 \ 1]^T\) and \(\hat{\mathbf{u}}(s) \in so(3)\) represents a skew symmetric matrix based on the components of the curvature vector \((\mathbf{u}(s) \in \mathbb{R}^3)\) that is in local coordinates. \(\mathbf{u}(s) = [\kappa_x(s) \ \kappa_y(s) \ 0]^T\), the last component is zero due to the assumption of no torsion. Frenet-Serret frames; the \(zt\)-axis is tangent to the center curve of the manipulator, the \(xt\)-axis is aligned with the inner normal and the \(yi\)-axis is aligned with the binormal vector (Figure 4).

B. Contact force estimation

The applied force at the tip of the manipulator is calculated using the constitutive relation and equations for equilibrium. The linear constitutive relation is as follows:
\[
\mathbf{m}(s) = \mathbf{R}(s) \mathbf{K}(s) \Delta \mathbf{u}(s),
\]
where \(\Delta \mathbf{u}(s) = \mathbf{u}^*(s) - \mathbf{u}(s)\), \(\mathbf{u}^*(s) \in \mathbb{R}^3\) is the curvature vector when no external force is applied and \(\mathbf{u}(s) \in \mathbb{R}^3\) is the curvature vector after the external force is applied on the manipulator, both are in local coordinates. \(\mathbf{K}(s) \in \mathbb{R}^{3 \times 3}\) is the stiffness matrix. The equations for equilibrium are:
\[
\begin{align*}
\mathbf{n}(s) &= \int_s^L \mathbf{f}(\sigma) \, d\sigma, \\
\mathbf{m}(s) &= \int_s^L \left[ \mathbf{r}(\sigma) \times \mathbf{f}(\sigma) \right] \mathbf{d\sigma} - \mathbf{r}(s) \times \mathbf{n}(s),
\end{align*}
\]
where \(\mathbf{f}(\sigma) \in \mathbb{R}^3\) is the external force at the tip of the manipulator and \(\sigma \in \mathbb{R}\) is a dummy integral variable. The force is modeled as a product of the unknown force \(\mathbf{F}_{ext} \in \mathbb{R}^3\) and a shifted Dirac delta function, \(\mathbf{f}(\sigma) = \mathbf{F}_{ext} \delta(\sigma - L)\), where \(L\) is the arc length at which the force is applied. Substituting \((17)\) and \((18)\) into \((19)\) leads to the estimate of the external force \((\mathbf{F}_{ext})\) that is given below:
\[
\mathbf{F}_{ext} = (\Delta \mathbf{r}(s))^T \mathbf{R}(s) \mathbf{K}(s) \Delta \mathbf{u}(s),
\]
where \((.)^T\) is the Moore-Penrose pseudoinverse and
\[
\Delta \mathbf{r}(s) = \mathbf{r}(L) - \mathbf{r}(s),
\]
\(\Delta \mathbf{r}(s) \in so(3)\) is a skew symmetric matrix based on the vector \((\Delta \mathbf{r}(s) \in \mathbb{R}^3)\) from \((21)\). The rotation matrix \(\mathbf{R}(s)\) in \((20)\) is calculated from \((16)\). Thus, \((20)\) gives the external force based on any point on the arc length and the evaluated parameters on the right hand side of the equation.

V. EXPERIMENTS AND RESULTS

This section presents the experiments used to validate the Rigid Link and Cosserat model, the experimental setup, the calibration procedures and results. 

A. Experimental Setup

The experimental setup consists of a continuum manipulator that is actuated by four tendons (DSM Dyneema B.V., Geleen, The Netherlands), shown in Figure 5(a). The backbone (Figure 5(b)) of the manipulator is made from a flexible Polyether ether ketone (PEEK) and it has grooves in which three optical fibers of diameter 250 \(\mu\text{m}\) are glued. Each fiber has 8 FBG sensors and the fibers are positioned such that corresponding FBG sensors are co-located (Figure 2). Thus, the backbone has 8 sets of co-located FBG sensors as shown in Figure 5(b). The setup has a linear stage to move the continuum manipulator along the global \(z\)-axis, a Deminsys Python FBG interrogator (Technobis group, Alkmaar, The Netherlands) and a Nano-43 6-DOF force/torque sensor (ATI...
Industrial Automation, Apex, USA). The actuation motors for the manipulator are Maxon EC-max 283840 (Maxon motor AG, Sachseln, Switzerland) and they are driven by Elmo controllers (Elmo Motion Control Ltd., Petach-Tikva, Israel). Controller Area Network (CAN) is used to provide communication with the interrogator and the force sensor.

**B. Calibration**

In order to accurately calculate curvature from the measured strains, the exact distance of the fiber from the center of the manipulator at the location of the sensor needs to be known. The shape sensing rod is placed in several constant curvature slots, which are laser cut in an acrylic plate (Figure 5(c)). Each fiber is separately calibrated by aligning the fiber with the bending direction of the rod. Using the curvature of the slot and the measured strain, the distance \( r_a, r_b \) and \( r_c \), (Figure 2) between the fiber and the center of the rod at each sensor location is calculated. The average value for all slots is calculated, and is used as the calibrated distance (Table I). These values are used to calculate the curvature vectors from the strain measurements, as described in Section II.

The flexural stiffness \( (K_{b,i} \text{ in (4)}) \) required by the Rigid Link model and the stiffness matrix \( (K(s) \text{ in (17)}) \), required by the Cosserat rod model are determined experimentally due to unavailability of accurate material properties. Data from 14 experiments in conjunction with lsqnonlin (MATLAB R2015b, The MathWork Inc., Natick, MA) is utilized to solve for the stiffness parameters. The remaining experiments were used to validate the models.

**C. Experiments**

The Rigid Link and the Cosserat models are validated using a tendon-driven continuum manipulator (Figure 5(a)). An external force \( (F_{ext}) \) is applied to the tip of the manipulator from three directions \( (\alpha_{ij}) \) which are 0°, 90° and 180° with respect to the global x-axis (Figure 5). The manipulator tip is tethered to the force sensor and the sensor is manually placed such that the tether is in-line with one of the three directions. Once the sensor is placed, the tension in the tether is increased, which results in an external force at the manipulator tip in the \( \alpha_{ij} \) direction. The experiment is repeated 10 times for each direction and the measurements from the FBG sensors and the force sensor are collected. Inputs to the two models are the measurements from the FBG sensors and the output is the tip force estimate. In the next sub-section, the force sensor measurement is compared to the force estimate from both models.

**D. Results**

The magnitude of the force from the sensor and the models are compared for all experiments. The plot of the force measured and the force estimated from a representative experiment is presented in Figure 6. It shows that both models can track the change in applied force. The mean error \( (\bar{e}) \) and the mean relative error \( (\bar{re}) \) as defined in (22) and (23) are reported in Table II.

\[
e(t) = \frac{\left| \left| \mathbf{F}_{\text{sen}}(t) \right| - \left| \mathbf{F}_{\text{mdl}}(t) \right| \right|}{\left| \mathbf{F}_{\text{sen}}(t) \right|}, \quad (22)
\]

\[
re(t) = \frac{e(t)}{\left| \mathbf{F}_{\text{sen}}(t) \right|} \quad s.t. \quad \left| \mathbf{F}_{\text{sen}}(t) \right| > 0, \quad (23)
\]

where \( t \in \mathbb{R} \) represents time, \( \mathbf{F}_{\text{sen}}(t) \in \mathbb{R}^3 \) is the force measurement from the sensor and \( \mathbf{F}_{\text{mdl}}(t) \in \mathbb{R}^3 \) is the force estimate from the models.
for accurate manipulation. The estimated forces could be used for closed loop respect. For future work, the wrenches along the shaft continuum manipulator. The results show that the Rigid Link model has a smaller error compared to the Cosserat model and it is computationally less complex.

This paper provides a framework for the FBG sensors that can be utilized for simultaneous shape and force sensing in continuum manipulators. In addition, two models for force sensing are presented and validated on a tendon driven continuum manipulator. The results show that the Rigid Link and Cosserat models can estimate the applied tip forces with an error of 11.2 mN (6.9%) and 15.9 mN (8.3%), respectively. For future work, the wrenches along the shaft of the manipulator and the axial force will be included in the models. The estimated forces could be used for closed loop force control coupled with other clinical imaging modalities for accurate manipulation.

### REFERENCES


